

# Mathematics and Visualization

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# Tutorials on Multiresolution in Geometric Modelling

Summer School Lecture Notes

With 172 Figures, 4 in Color



Springer

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# Preface

Multiresolution techniques in geometric modelling and computer graphics are concerned with the generation, representation, and manipulation of geometric objects at different levels of detail. In this rapidly developing research field both mathematics and computer science play an important role. The wide range of applications includes fast visualization and rendering, as well as coding, compression and digital transmission of 3D geometric objects, such as freeform surfaces and terrain models.

This book is based on tutorial lectures presented during the European Summer School *Principles of Multiresolution in Geometric Modelling*, held at the Munich University of Technology, Germany, August 22-30, 2001, see the web page [www.ma.tum.de/primus2001/](http://www.ma.tum.de/primus2001/). The summer school offered an interactive training course aimed at graduate students and researchers with little or no prior knowledge of multiresolution techniques. The lectures were given by international experts, covering important aspects and recent developments in this research field. The summer school was attended by 127 participants from 22 countries.

The school was organized as part of the current European Research and Training Network *Multiresolution in Geometric Modelling* (MINGLE), see [www.oslo.sintef.no/mingle/](http://www.oslo.sintef.no/mingle/), comprising nine partners from six countries. The MINGLE project is currently offering research training fellowships to young European researchers hosted by the partners.

We have arranged the thirteen chapters of the book in four parts:

**Part I.** *Subdivision;*

**Part II.** *Wavelets;*

**Part III.** *Scattered Data Modelling;*

**Part IV.** *Coding and Data Structures.*

Part I gives an introduction to the theory of constructing curves and surfaces by subdivision schemes. Part II discusses wavelet techniques using nonuniform splines, non-nested scaling functions, and spherical harmonics. Part III covers radial basis functions, bivariate splines on triangulations, and parameterization techniques for 3D point clouds and triangulations. Finally, Part IV presents simplification and compression techniques for 3D meshes and multiresolution data structures.

All texts were designed as tutorial introductions to the selected topics. Corresponding exercises support this tutorial character. Hints and solutions may be found in some of the texts, while others are supplemented by material and software which can be downloaded from the summer school's web site [www.ma.tum.de/primus2001/](http://www.ma.tum.de/primus2001/).

In order to ensure the scientific and instructive quality of the contributions to this book, each chapter was carefully refereed by three different types of anonymous reviewers: one expert external to MINGLE, one expert from the MINGLE consortium, and one young researcher, who attended the summer school.

Finally, we wish to thank those who supported the summer school and the making of this book. First and foremost it is a pleasure to acknowledge the generous support and hospitality granted by the Munich University of Technology. In addition, partial financial support was given by SINTEF Applied Mathematics, and the European Commission, through the MINGLE network (contract no. HPRN-CT-1999-00117). Moreover, special thanks go to all contributors and referees, as well as to the participants of the summer school. Last but not least, the friendly and effective collaboration with Springer-Verlag, Heidelberg, through Martin Peters, Ruth Allewelt, Leonie Kunz, and Ute McCrory is kindly appreciated.

*Oslo, March 2002*

*Armin Iske  
Ewald Quak  
Michael S. Floater*

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