

Accelerator Physics

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Levi Schächter

Beam–Wave Interaction in Periodic and Quasi-Periodic Structures

With 109 Figures



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To my parents

Preface

During the past seven years I have been involved in the investigation of high-power microwave sources for accelerator and radar applications. As for many others before me, the starting point of this book was a collection of notes on theoretical topics out of the material I had been working on. The notes were the core of a course for graduate students at Cornell University. When I started to prepare these notes it seemed a fairly straight-forward and not very time-consuming task since I had most of the material well organized. Today, three years after the preparation of the first notes, I can only wonder how naive this thought was.

Most of my work was oriented towards analytic and quasi-analytic techniques for the investigation of the interaction of an electron beam with electromagnetic waves. These topics are presented in Chaps. 4 and 6. However, for a systematic elaboration of these topics it was necessary to provide some general background, therefore parts of what are today Chaps. 2, 3, and 5 were prepared. Related topics of acceleration concepts were also prepared to some extent but I ran out of time and the material (Chap. 8) was not delivered. In the meantime, various sections of this book were taught at the Technion-Israel Institute of Technology and Ben-Gurion University. In the last version I included a discussion on free electron lasers (Chap. 7).

In this book I present primarily models of the various concepts of beam-wave interactions with emphasis on analytic and quasi-analytic methods. These have to be the basis of any numerical simulation which is today the elementary design tool of any advanced radiation source. A model is an idealization of what we conceive as the real system. In practice, this idealization manifests itself as mathematical approximations that if not revealed and/or evaluated, may lead to a non-realistic analysis and therefore to wrong conclusions. Therefore I believe the reader should be given all reasonable details for tracing each step in the analysis. She or he has to decide whether to skip a certain step because it is trivial or because the implications to the model are obvious. For this reason detailed analysis is used in most of the cases.

Many people have contributed directly or indirectly to this book and I wish to thank them all. The collaboration with Prof. J.A. Nation of Cornell University, his encouragement and critique, had a crucial impact on the material presented here and he deserves my sincere gratitude. I benefited from

VIII Preface

enlightening discussions with Prof. Norman M. Kroll of the University of California at San Diego. I wish to thank Dr. G. S. Kerslick and Mr. J. D. Ivers who contributed hours and days in discussions on many of the topics presented here. Special acknowledgement is deserved by those who were, or still are, graduate students and carried out the experiments that, in turn, were either the cause or the result of the concepts developed and presented here. They are: Dr. D.A. Shiffler, Dr. T.J. Davis, Dr. E. Kuang, Dr. S. Naqvi and Mr. D. Flechtner. Most of this research was supported by the United States Department of Energy. During the last three years I also benefited from the support of the Bi-National United States - Israel Science Foundation.

Last but by no means least I want to thank my wife Tal, and my children Michal and Roy who endured the long hours of work with patience and love.

Haifa, August 1996

Levi Schächter

Table of Contents

1. Introduction	1
1.1 Single-Particle Interaction	3
1.1.1 Infinite Length of Interaction	3
1.1.2 Finite Length of Interaction	4
1.1.3 Finite Length Pulse	5
1.1.4 Cerenkov Interaction	6
1.1.5 Compton Scattering: Static Fields	7
1.1.6 Compton Scattering: Dynamic Fields	8
1.1.7 Uniform Magnetic Field	9
1.1.8 Synchronism Condition	10
1.2 Radiation Sources: Brief Overview	11
1.2.1 The Klystron	11
1.2.2 The Traveling Wave Tube	13
1.2.3 The Gyrotron	14
1.2.4 The Free Electron Laser	16
1.2.5 The Magnetron	17
1.2.6 The Vircator	18
1.2.7 The Magnicon	18
1.2.8 Classification Criteria	20
1.3 Choice and Organization of the Material	22
2. Elementary Electromagnetic Phenomena	27
2.1 Maxwell's Equations	27
2.1.1 Constitutive Relations	28
2.1.2 Boundary Conditions	29
2.1.3 Poynting's Theorem	29
2.1.4 Steady-State Regime	31
2.1.5 Complex Poynting's Theorem	32
2.1.6 Potentials	33
2.2 Simple Wave Phenomena	34
2.2.1 Simple Propagating Waves	34
2.2.2 The Radiation Condition	35
2.2.3 Evanescent Waves	36
2.2.4 Waves of a Moving Charge	37

2.3	Guided Waves	38
2.3.1	Transverse Electromagnetic Mode	38
2.3.2	Transverse Magnetic Mode	40
2.3.3	Velocities and Impedances	42
2.3.4	Transverse Electric Mode	45
2.4	Green's Scalar Theorem	48
2.4.1	Cerenkov Radiation in the Boundless Case	49
2.4.2	Cerenkov Radiation in a Cylindrical Waveguide	51
2.4.3	Cerenkov Force	55
2.4.4	Ohm Force	58
2.5	Finite Length Effects	60
2.5.1	Impedance Discontinuities	61
2.5.2	Geometric Discontinuity	63
2.5.3	Wake-Field in a Cavity	70
	Exercises	75
3.	Elementary Electron Dynamics	77
3.1	Classical Dynamics	77
3.1.1	Newtonian Equations of Motion	78
3.1.2	Lagrangian Formalism	79
3.1.3	Hamiltonian Formalism	80
3.1.4	Kinetic Approximation: Liouville's Theorem	82
3.1.5	Hydrodynamic Approximation	84
3.1.6	Global Energy Conservation	86
3.2	Special Theory of Relativity	87
3.2.1	Principles	87
3.2.2	Lorentz Transformation	88
3.2.3	Phase Invariance	89
3.2.4	Field Transformation	92
3.3	Electron Generation	94
3.3.1	Child-Langmuir Limiting Current	95
3.3.2	Beyond Child-Langmuir Limit: Ferro-Electric Emission	99
3.4	Beam Propagation	102
3.4.1	Beam Propagation in Free Space: Uniform B-Field ..	103
3.4.2	Beam Propagation in Free Space: Periodic B-Field ..	106
3.4.3	Beam Propagation in a Uniform Waveguide	108
3.4.4	Beam Propagation in a Disk-Loaded Waveguide	111
3.4.5	Beam Emittance and Brightness	116
3.5	Space-Charge Waves	118
3.5.1	Slow and Fast Space-Charge Waves	120
3.5.2	"Negative" and "Positive" Energy	120
3.5.3	Resistive Wall Instability	121
3.5.4	Two-Beam Instability	124

3.5.5	Interference of Space-Charge Waves	127
Exercises	131
4.	Models of Beam-Wave Interaction in Slow-Wave Structures	133
4.1	Semi-Infinite Structure: Pierce-Like Theory	134
4.1.1	Dielectric Filled Waveguide	135
4.1.2	Partially Filled Waveguide	141
4.2	Finite Length Effects	146
4.2.1	Oscillator	147
4.2.2	Gain and Bandwidth Considerations	149
4.2.3	Interaction in an Extended Cavity	153
4.2.4	Backward-Wave Oscillator	155
4.3	Macro-Particle Approach	157
4.3.1	Simplified Set of Equations	157
4.3.2	Phase-Space Distribution: Linear Regime	161
4.3.3	Phase-Space Distribution: Saturation	165
4.3.4	Interaction in a Slowly Tapered Structure	166
4.3.5	Noise	167
4.3.6	Super-Radiant Emission	169
4.3.7	Resonant Particle Model	170
4.4	Complex Interaction Impedance	174
4.5	Amplifier and Oscillator: A Unified Approach	178
4.5.1	Simplified Set of Equations	179
4.5.2	Ideal Amplifier	183
4.5.3	Ideal Oscillator	183
4.5.4	Global Energy Conservation	185
4.5.5	Reflections in an Amplifier	186
4.5.6	Spatial Variations in an Oscillator	189
Exercises	193
5.	Periodic Structures	195
5.1	The Floquet Theorem	196
5.2	Closed Periodic Structure	201
5.2.1	Dispersion Relation	203
5.2.2	Spatial Harmonics Coupling	206
5.2.3	Interaction Parameters	208
5.3	Open Periodic Structure	213
5.3.1	Dispersion Relation	214
5.3.2	Interaction Parameters	217
5.3.3	Green's Function: The Smith-Purcell Effect	220
5.3.4	Scattering from Cylindrical Gratings	228
5.4	Transients	230
Exercises	233

6. Quasi-periodic Structures	235
6.1 Homogeneous Solution	237
6.1.1 Definition of the Model	238
6.1.2 Evaluation of Green's Function	240
6.1.3 Transmission and Reflection	241
6.2 Non-homogeneous Solution	243
6.2.1 Green's Function	243
6.2.2 Stationary Dipole	246
6.2.3 Distributed Current Density	247
6.3 Beam-Wave Interaction: Hydrodynamic Approximation	251
6.3.1 Definition of the Model	251
6.3.2 Evaluation of Green's Function	253
6.3.3 Transmission and Reflection	256
6.4 Macro-Particle Approach	259
6.4.1 Definition of the Model	260
6.4.2 Evaluation of Green's Function	262
6.4.3 The Governing Equations	265
6.4.4 Qualitative Approach	270
Exercises	272
7. Free-Electron Laser	273
7.1 Spontaneous Radiation	274
7.2 Low-Gain Compton Regime	280
7.3 High-Gain Compton Regime	283
7.3.1 The Dispersion Relation	285
7.3.2 Cold Beam Operation	287
7.3.3 Warm Beam Operation	291
7.4 Macro-Particle Approach	292
7.4.1 Basic Formulation	293
7.4.2 Resonant Particle Solution	297
7.4.3 Buckets	298
7.4.4 Energy Spread	302
7.5 Other FEL Schemes	307
7.5.1 Gas Loaded FEL	307
7.5.2 Longitudinal Wiggler FEL	308
7.5.3 Rippled-Field Magnetron	308
7.5.4 Wiggler and Guiding Magnetic Field	309
7.5.5 Electromagnetic Wiggler	309
7.5.6 Electrostatic Wiggler	310
7.5.7 Channeling Radiation	312
Exercises	313
8. Basic Acceleration Concepts	315
8.1 Linear Accelerator Concepts	316

8.1.1	Constant Gradient and Constant Impedance Structures	316
8.1.2	Auxiliary Coupling	320
8.1.3	Phase Dynamics	321
8.1.4	Transverse Effects: Panofsky-Wenzel Theorem	324
8.2	Advanced Accelerator Concepts: Brief Overview	328
8.3	Wake-Field Accelerator	329
8.3.1	Dielectric Wake-Field Accelerator	329
8.3.2	Periodic Structure Wake-Field Accelerator	331
8.3.3	Laser Wake-Field Accelerator	332
8.4	Plasma Beat-Wave Accelerator	333
8.5	Inverse of Radiation Effects	334
8.5.1	Inverse FEL	334
8.5.2	Inverse Cerenkov	335
8.5.3	Open Structure Accelerator	336
8.5.4	PASER: Particle Acceleration by Stimulated Emission of Radiation	337
8.6	Two-Beam Accelerator	341
	Exercises	342
	References	345
	Subject Index	353