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Karin Möllering

Inventory Rationing

A New Modeling Approach Using Markov
Chain Theory

Karin Möllering
Wiesbaden, Germany

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*Zwei Dinge sind zu unserer Arbeit notwendig:
Unermüdliche Ausdauer und die Bereitschaft,
etwas, in das man viel Zeit und Arbeit gesteckt
hat, wieder wegzwerfen.*

Albert Einstein

Friends of mine once said about their time reading for a PhD, "If I had known what it cost me, I would never have started" and characterized the four years as the honeymoon, in which everything is exciting and you are overconfident, the panicking year in which you realize that you have not done a lot yet and are no longer sure whether you will be able to deliver, the third year during which you will work very hard and the rewarding year at the end. Now that I have finished my degree I know how much truth those words contained. And I would like to take the opportunity to thank all those who stood up to me throughout the last four years.

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Cologne, January 2007

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List of Abbreviations

ADI	advanced demand information
AP	aggregation policy
AV	averaging policy
c.d.f.	cumulative distribution function
cf.	confer
CPU	central processing unit
CRP	constant rationing policy
cts.	continuous
ed.	editor, edition
e.g.	exempli gratia
et al.	et alii
FCFS	first come, first served
GDP	gross domestic product
HP	Hewlett-Packard
i.i.d.	independent and identically distributed
i.e.	id est
LCL	linear critical level policy
NPO	next period optimization policy
p.d.f.	probability density function
per.	periodic
pp.	pages

resp.	respectively
r.v.	random variable
SP	separation policy
vs.	versus
WIP	work-in-process
w.r.t.	with respect to

List of Symbols

α, α_i	required α -service level (of class i)
$\bar{\alpha}$	demand-weighted average α -service level
β, β_i	required β -service level (of class i)
B_t, B_{it}	(total) backorders (of class i) in period t
B, B_i	(total) backorders (of class i) in steady state
B_i^{XY}	class i backorders in steady state under policy XY
CL, CL_k	critical level (on diagonal k)
CL_t^{XY}	critical level in period t under policy XY
CL^{XY}	expected critical level under policy XY
CL_i^S	minimal critical level satisfying the service constraint for class i given order-up-to level S
c_o, c_u	unit overage and underage cost per period
D_t, D_{it}	(total) demand (of class i) in period t
D, D_i	(total) demand (of class i) in steady state
d	period of a Markov chain
ETC, ETC^{XY}	expected total cost per period (under policy XY)
$EX, E[X]^+$	expectation of the (positive part of the) r.v. X
$f_{ij}^{(n)}$	probability to enter state j starting in state i for the first time after n steps
$F, F^{-1}, \bar{F}$	cumulative distribution function, its inverse, $1 - F$
h	unit holding cost per period

$\mathfrak{S}$	state space of a Markov chain
I_t, I	on-hand inventory after order arrival at the beginning of period t , of a period in steady state
IH_t, IH, IH^{XY}	on-hand inventory at the end of period t , of a period in steady state (under policy XY)
IO_t, IO	inventory on order at the beginning of period t , of a period in steady state
κ	constant of the linear critical level policy
λ_i	parameter of the Poisson distribution (demand of class i)
LT	lead time
m_{ii}	expected return time of state i
N	truncation value
ν	weight factor of the linear critical level policy
p, p_i	unit penalty cost per period (of class i)
$\bar{p}$	demand-weighted average period unit penalty cost
$P, P^{(n)}$	(n -step) transition matrix
$p_{ij}, p_{ij}^{(n)}$	(n -step) transition probability from state i to j
π_x, π_x^N	invariant probability of state x (with transition matrix truncated at N)
$(\pi_x)_{x \in \mathfrak{S}}, (\pi_x^N)_{x \in \mathfrak{S}}$	invariant distribution (with matrix truncated at N)
Q, Q_t	order quantity (in period t)
R	review period
s	reorder point
S, S^{XY}	order-up-to level (under policy XY)
S_i^0	Minimal order-up-to level to meet class i service constraint if $CL = 0$
T	index of the Markov chain, here $T = \mathbb{N}$
X_t	state of a Markov chain in period t , also: random variable

Z_k^{CL}	expected total cost with order-up-to level $S = k + CL$ and critical level CL
Ω	set of potentially optimal parameter constellations
$\leq_{st}, \geq_{st}$	stochastically smaller resp. larger

If necessary, we use superscripts $\cdot^{S,CL}$ to indicate dependencies on the decision variables S and CL . The superscript $\cdot^*$ indicates optimality. Specific realizations of random variables are denoted by small letters.