Sylvestre Gallot
Dominique Hulin
Jacques Lafontaine

Riemannian Geometry

Second Edition
With 95 Figures

Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo
Hong Kong Barcelona

Sylvestre Gallot
Ecole Polytechnique, Centre de Mathématiques
Unité de Recherche Associée du CNRS D 0169
F-91128 Palaiseau Cedex, France

Dominique Hulin
Ecole Polytechnique, Centre de Mathématiques
Unité de Recherche Associée du CNRS D 0169
F-91128 Palaiseau Cedex, France

Jacques Lafontaine
Université de Montpellier
Département de Mathématiques
GETODIM - Unité de Recherche Associée du CNRS 1407
F-34095 Montpellier Cedex 5, France

Mathematics Subject Classification (1980): 53-XX, 58-XX

ISBN-13:978-3-540-52401-4 e-ISBN-13:978-3-642-97242-3
DOI:10.1007/978-3-642-97242-3

2141/3140-543210 - Printed on acid-free paper

PREFACE TO THE SECOND EDITION

In this second edition, the main additions are a section devoted to surfaces with constant negative curvature, and an introduction to conformal geometry.

Also, we present a -soft- proof of the Paul Levy-Gromov isoperimetric inequality, kindly communicated by G. Besson.

Several people helped us to find bugs in the first edition. They are not responsible for the persisting ones! Among them, we particularly thank Pierre Arnoux and Stefano Marchiafava.

We are also indebted to Marc Troyanov for valuable comments and suggestions.

INTRODUCTION

This book is an outgrowth of graduate lectures given by two of us in Paris. We assume that the reader has already heard a little about differential manifolds. At some very precise points, we also use the basic vocabulary of representation theory, or some elementary notions about homotopy. Now and then, some remarks and comments use more elaborate theories. Such passages are inserted between *.

In most textbooks about Riemannian geometry, the starting point is the local theory of embedded surfaces. Here we begin directly with the so-called "abstract" manifolds. To illustrate our point of view, a series of examples is developed each time a new definition or theorem occurs. Thus, the reader will meet a detailed recurrent study of spheres, tori, real and complex projective spaces, and compact Lie groups equipped with bi-invariant metrics. Notice that all these examples, although very common, are not so easy to realize (except the first) as Riemannian submanifolds of Euclidean spaces.

The first chapter is a quick introduction to differential manifolds. Proofs are often supplied with precise references. However, numerous examples and exercises will help the reader to get familiar with the subject.

Chapters II and III deal with basic Riemannian geometry of manifolds, as described in the content table. We finish with global results (Cartan-Hadamard, Myers' and Milnor's theorems) concerning relations between curvature and topology. By the way, we did not resist the temptation to give an overview of recent research results. We hope the reader will want to look at the original papers.

Analysis on manifolds has become a wide topic, and chapter IV is only an introduction. We focused on the Weitzenbock formula, and on some aspects of spectral theory. Our Ariadne's thread was what our "Mentor" Marcel Berger calls "la domination universelle de la courbure de Ricci", discovered by Gromov in the seventies. Chapter V deals with more classical topics in Riemannian submanifolds.

The reader will find numerous exercises. They should be considered as a part of the text. This is why we gave the solutions of most of them.

SOME HISTORICAL AND HEURISTIC REFERENCES

Riemannian Geometry is indeed very lively today, but most basic notions go back to the nineteenth century.

At the beginning of each chapter, we endeavored to explain what is going on. But these short introductions do not replace serious history or popularization works. We recommend the following, to be read concurrently with our book:

E. Cartan : La Géométrie des espaces de Riemann [Ca];

M. Spivak : Differential geometry (t.2) [Sp];

P. Dombrowski : 150 years after Gauss' "disquitiones generales circa superficies curvas" [Dom];

M. Berger : La géométrie métrique des variétés riemanniennes [Br3];

J.P. Bourguignon : Géométrie et Physique [Bn];

S. Gallot : Géométries [Ga5].

ACKNOWLEDGEMENTS

During the preparation of this book, the help of our host institutions, the Institut Fourier of Grenoble (S.G.), the Centre de Mathématiques de l'Ecole Polytechnique (D.H.), and the Unité Associée au C.N.R.S. no. 212 (J.L.), was decisive. At the Ecole Polytechnique, Thomas Ehrhardt taught us the wonderful TEX system. Particular thanks are due to him.

CONTENTS

Chapter I. Differential Manifolds