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Riemannian Geometry

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INTRODUCTION

This book is an outgrowth of graduate lectures given by two of us in Paris. We assume that the reader has already heard a little about differential manifolds. At some very precise points, we also use the basic vocabulary of representation theory, or some elementary notions about homotopy. Now and then, some remarks and comments use more elaborate theories. Such passages are inserted between *.

In most textbooks about Riemannian geometry, the starting point is the local theory of embedded surfaces. Here we begin directly with the so-called "abstract" manifolds. To illustrate our point of view, a series of examples is developed each time a new definition or theorem occurs. Thus, the reader will meet a detailed recurrent study of spheres, tori, real and complex projective spaces, and compact Lie groups equipped with bi-invariant metrics. Notice that all these examples, although very common, are not so easy to realize (except the first) as Riemannian submanifolds of Euclidean spaces.

The first chapter is a quick introduction to differential manifolds. Proofs are often supplied with precise references. However, numerous examples and exercises will help the reader to get familiar with the subject.

Chapters II and III deal with basic Riemannian geometry of manifolds, as described in the content table. We finish with global results (Cartan-Hadamard, Myers' and Milnor's theorems) concerning relations between curvature and topology. By the way, we did not resist the temptation to give an overview of recent research results. We hope the reader will want to look at the original papers.

Analysis on manifolds has become a wide topic, and chapter IV is only an introduction. We focused on the Weitzenbock formula, and on some aspects of spectral theory. Our Ariadne's thread was what our "Mentor" Marcel Berger calls "la domination universelle de la courbure de Ricci", discovered by Gromov in the seventies. Chapter V deals with more classical topics in Riemannian submanifolds.

The reader will find numerous exercises. They should be considered as a part of the text. This is why we gave the solutions of most of them.

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