

H. Haken Synergetics—An Introduction



Hermann Haken

Synergetics

An Introduction

Nonequilibrium Phase Transitions and Self-Organization
in Physics, Chemistry and Biology

With 125 Figures

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To the Memory of

Maria and Anton Vollath

Preface

The spontaneous formation of well organized structures out of germs or even out of chaos is one of the most fascinating phenomena and most challenging problems scientists are confronted with. Such phenomena are an experience of our daily life when we observe the growth of plants and animals. Thinking of much larger time scales, scientists are led into the problems of evolution, and, ultimately, of the origin of living matter. When we try to explain or understand in some sense these extremely complex biological phenomena it is a natural question, whether processes of self-organization may be found in much simpler systems of the unanimated world.

In recent years it has become more and more evident that there exist numerous examples in physical and chemical systems where well organized spatial, temporal, or spatio-temporal structures arise out of chaotic states. Furthermore, as in living organisms, the functioning of these systems can be maintained only by a flux of energy (and matter) through them. In contrast to man-made machines, which are devised to exhibit special structures and functionings, these structures develop spontaneously—they are self-organizing. It came as a surprise to many scientists that numerous such systems show striking similarities in their behavior when passing from the disordered to the ordered state. This strongly indicates that the functioning of such systems obeys the same basic principles. In our book we wish to explain such basic principles and underlying conceptions and to present the mathematical tools to cope with them.

This book is meant as a text for students of physics, chemistry and biology who want to learn about these principles and methods. I have tried to present mathematics in an elementary fashion wherever possible. Therefore the knowledge of an undergraduate course in calculus should be sufficient. A good deal of important mathematical results is nowadays buried under a complicated nomenclature. I have avoided it as far as possible though, of course, a certain number of technical expressions must be used. I explain them wherever they are introduced. Incidentally, a good many of the methods can also be used for other problems, not only for self-organizing systems. To achieve a self-contained text I included some chapters which require some more patience or a more profound mathematical background of the reader. Those chapters are marked by an asterisk. Some of them contain very recent results so that they may also be profitable for research workers.

The basic knowledge required for the physical, chemical and biological systems is, on the average, not very special. The corresponding chapters are arranged in such a way that a student of one of these disciplines need only to read “his” chapter. Nevertheless it is highly recommended to browse through the other

chapters just to get a feeling of how analogous all these systems are among each other. I have called this discipline “synergetics”. What we investigate is the joint action of many subsystems (mostly of the same or of few different kinds) so as to produce structure and functioning on a macroscopic scale. On the other hand, many different disciplines cooperate here to find general principles governing self-organizing systems.

I wish to thank Dr. Lotsch of Springer-Verlag who suggested writing an extended version of my article “Cooperative phenomena in systems far from thermal equilibrium and in nonphysical systems”, in *Rev. Mod. Phys.* (1975). In the course of writing this “extension”, eventually a completely new manuscript evolved. I wanted to make this field especially understandable to students of physics, chemistry and biology. In a way, this book and my previous article have become complementary.

It is a pleasure to thank my colleagues and friends, especially Prof. W. Weidlich, for many fruitful discussions over the years. The assistance of my secretary, Mrs. U. Funke, and of my coworker Dr. A. Wunderlin was an enormous help for me in writing this book and I wish to express my deep gratitude to them. Dr. Wunderlin checked the formulas very carefully, recalculating many of them, prepared many of the figures, and made valuable suggestions how to improve the manuscript. In spite of her extended administrative work, Mrs. U. Funke has drawn most of the figures and wrote several versions of the manuscript, including the formulas, in a perfect way. Her willingness and tireless efforts encouraged me again and again to complete this book.

Hermann Haken

Stuttgart, November 1976

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