Ergebnisse der Mathematik und ihrer Grenzgebiete

Band 37

Herausgegeben von
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Redaktion: P. J. Hilton

Varieties of Groups

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Springer-Verlag New York Inc. 1967

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ISBN 978-3-642-88601-0 ISBN 978-3-642-88599-0 (eBook)
DOI 10.1007/978-3-642-88599-0

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© by Springer-Verlag, Berlin · Heidelberg 1967
Softcover reprint of the hardcover 1st edition 1967
Library of Congress Catalog Card Number 67 – 15 607

Title No. 4581

To Bernhard

Preface

Varieties of algebras are equationally defined classes of algebras, or "primitive classes" in MAL'CEV's terminology. They made their first explicit appearance in the 1930's, in Garrett BIRKHOFF's paper on "The structure of abstract algebras" and B.H. NEUMANN's paper "Identical relations in groups I". For quite some time after this, there is little published evidence that the subject remained alive. In fact, however, as part of "universal algebra", it aroused great interest amongst those who had access, directly or indirectly, to PHILIP HALL's lectures given at Cambridge late in the 1940's. More recently, category theory has provided a general setting since varieties, suitably interpreted, are very special examples of categories. Whether their relevance to category theory goes beyond this, I do not know. And I doubt that the category theoretical approach to varieties will be more than a fringe benefit to group theory. Whether or not my doubts have substance, the present volume owes its existence not to the fact that varieties fit into a vastly more general pattern, but to the benefit group theory has derived from the classification of groups by varietal properties. It is this aspect of the study of varieties that seems to have caused its reappearance in the literature in the 1950's. Since then varietal methods in group theory have rapidly gained impetus and, as is wont to happen, they have thrown up their own problems and developed into a branch of group theory which is as much of intrinsic interest as it is powerful as a tool.

A course of lectures given in 1963 at the Manchester College of Science and Technology started the process of gathering and sorting the results on varieties of groups. Perhaps this very process has had some share in accelerating progress. In any event, when, after too long a time, the manuscript was ready for the publishers, it was also ready to be re-written. Experience shows that a report of this kind, delayed too long, becomes useless (if, indeed, it appears at all!). I therefore decided against re-writing which would have meant a long delay. It may then be in order to indicate briefly in this preface two of the points that I would have wanted to attend to.

The first is mainly, though not entirely, a matter of organization of material. Knowledge of metabelian varieties was scant until quite recently when it expanded so rapidly that, to do it justice, a separate chapter should now have been given to this topic. Instead I have merely added to the information provided in various contexts – a plan that was natural and adequate a short time ago. I have tried to ensure that these scattered references to the metabelian case are easily located by means of the index.

Secondly, I want to draw attention to an alternative development which has great advantages. ŠMEL'KIN'S embedding theorem (22.48) should, I believe, be made the starting point of the treatment of product varieties. Its proof is direct, needing no more than basic facts and the construction of the verbal wreath product; but its use would shorten and simplify much of Chapter 2 and several other topics besides (for example in Chapter 4).

This report would not have been written but for the interest and active participation of my audiences in the lectures at Manchester, mentioned earlier, and in similar lectures given at the Australian National University, Canberra. JOHN P. COSSEY and IAN D. MACDONALD checked parts of the manuscript; so did L.G. KOVÁCS, and M.F. NEWMAN read most of it at some stage or other. KOVÁCS and NEWMAN contributed much more in the way of elucidation, simplification and correction than can be apparent from the text. B.H. NEUMANN and PETER M. NEUMANN read the proofs and, aven at that late stage, prevented a number of minor and major accidents. I wish to acknowledge in particular the efforts of the latter who seemed to interpret "proof reading" to mean "checking all proofs". I record here my gratitude to all of them, and affirm, as is usual but still important, that any errors that remain are mine. Last, but not least, I acknowledge SPRINGER'S cooperation that was indeed all that one has come to expect of this name.

Canberra, November 1966 HANNA NEUMANN

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Note

The numbering is decimal; thus for example the statement numbered 34.12 refers to Chapter 3, Section 4, Subsection 1, Item 2. Subsections are not formally marked in any other way, and a subsection may contain no more than a single numbered item – such as 32.1.

The presentation is intended to be self-contained. Facts have been used without proof only if they are "known" in the sense that they have found their way into at least one textbook. For less familiar facts of this kind – here the choice is of necessity subjective – a textbook reference is given explicitly, and referring to one textbook does not mean that others may not also serve. Apart from this statements without proof occur, I hope, only on the fringe of the main development, in examples and in summaries of further results.

The textbooks referred to are the following; any reference containing merely the author's name and a page or section number is to these books.

COHN, P.M.: Universal algebra. Harper's Series in Modern Mathematics. New York: Evanston and London: Harper & Row 1965.

CURTIS, C.W., and I.R. REINER: Representation theory of finite groups and associative algebras. New York and London: Interscience Publ. 1962.

HALL Jr., MARSHALL: The theory of groups. New York: Macmillan 1959.

KUROSH, A.G.: The theory of groups, vol. I and II. Translat. from the Russian and edit. by K.A. HIRSCH, 1st or 2nd ed. New York: Chelsea 1956, 1960.

SCOTT, W.R.: Group theory. Englewood Cliffs, New Jersey: Prentice Hall Inc. 1964.

Since this report went to press, the book by W. MAGNUS, A. KARRASS and D. SOLITAR, Combinatorial Group Theory, Interscience has appeared. Had it been available earlier, reference to it would have been frequent, for example wherever commutator calculus is used.