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# Approximation of Functions: Theory and Numerical Methods

Translated by Larry L. Schumaker

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Approximation von Funktionen und ihre Numerische Behandlung.
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## Preface to the first German edition

It has only been in the past few years that those parts of approximation theory which can be applied to numerical problems have been strongly developed. The idea of obtaining a (in some sense) best approximation of a function gained considerable importance with the application of electronic computers. Some of the theoretical fundamentals necessary for practical problems can be found scattered about in a few books. However, by far the greatest portion of the theoretical and practical investigations can be studied only in the original papers. This provides the purpose of this book: to collect essential results of approximation theory which on the one hand makes possible a fast introduction to the modern development of this area, and on the other hand provides a certain completeness to the problem area of Tchebycheff approximation — not to imply by any means that a comprehensive survey of the literature is attempted. The material has been chosen from the subjective standpoint of its importance for applications. This also applies, for example, to the asymptotic investigations of § 3, since I am of the opinion that even in numerical approximation some thought should at least be given to what asymptotic precision can be expected. I have confined myself almost exclusively to the theory of uniform approximation since it has by far the greatest practical importance.

Part I is concerned with linear approximation. Chapter 3 contains what at present must be considered as the shortest approach to the linear theory. The details of the classical case of polynomial approximation (§ 6) are not much known, and the approach to the results is often laborious, so that I have decided to give a complete exposition. A special chapter (§ 7) has been dedicated to numerical methods of linear approximation, while constructive methods for non-linear approximation have been included with the theory in the individual sections. The bulk of Part II is related to newer investigations which I have carried out with D. Schwedt. Here we develop a theory of non-linear approximation which can be applied to various numerical problems.

With a few exceptions, all of the theorems in normal type have been presented with proofs (partly new). References to further studies are set in small type.

Unfortunately, because of space limitations various aspects of approximation theory have been completely disregarded. This includes, for example, the so-called $L_p$ approximation, the Bernstein approximation problem (approximation on the real line by certain entire functions), and the highly interesting studies of J. L. WALSH on approximation in the complex plane.

I would like to extend sincere thanks to Professor L. COLLATZ for his many encouragements for the writing of this book. Thanks are equally due to Springer-Verlag for their ready agreement to my wishes, and for the excellent and competent composition of the book. In addition, I would like to thank Dr. W. KRABS, Dr. A.-G. MEYER and D. SCHWEDT for their very careful reading of the manuscript.

Hamburg, March 1964

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## Preface to the English Edition

This English edition was translated by Dr. LARRY SCHUMAKER, Mathematics Research Center, United States Army, The University of Wisconsin, Madison, from a supplemented version of the German edition. Apart from a number of minor additions and corrections and a few new proofs (e.g., the new proof of JACKSON'S Theorem), it differs in detail from the first edition by the inclusion of a discussion of new work on comparison theorems in the case of so-called regular Haar systems (§ 6) and on Segment Approximation (§ 11). I want to thank the many readers who provided comments and helpful suggestions. My special thanks are due to the translator, to Springer-Verlag for their ready compliance with all my wishes, to Mr. HELMUT UNTERSTEIN for his valuable help, and to Miss GUDRUN STECHER and Miss CHRISTEL FRANKE for their careful typing of the manuscript.

Clausthal-Zellerfeld, May 1967

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