Rolf Nevanlinna

Analytic Functions

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Phillip Emig

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Professor Dr. Rolf Nevanlinna
Academy of Finland

Phillip Emig, Ph. D.
Granada Hills, California (U.S.A.)

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Preface

The present monograph on analytic functions coincides to a larg extent with the presentation of the modern theory of single-value analytic functions given in my earlier works "Le théorème de Picarc Borel et la théorie des fonctions méromorphes" (Paris: Gauthier-Villai 1929) and "Eindeutige analytische Funktionen" (Die Grundlehren de mathematischen Wissenschaften in Einzeldarstellungen, Vol. 46, 1s edition Berlin: Springer 1936, 2nd edition Berlin-Göttingen-Heidelberg Springer 1953).

In these presentations I have strived to make the individual result and their proofs readily understandable and to treat them in the ligh of certain guiding principles in a unified way. A decisive step in thi direction within the theory of entire and meromorphic functions consiste in replacing the classical representation of these functions through ca nonical products with more general tools from the potential theor (Green's formula and especially the Poisson-Jensen formula). On thi foundation it was possible to introduce the quantities (the characteristic the proximity and the counting functions) which are definitive for th description of the asymptotic properties of an analytic function in th vicinity of essential singularities. At the same time they lead to far reaching extensions and sharpenings of Picard's theorem, in the directior of the general value distribution theory, which is concerned with th distribution and density of those points at which an analytic functior assumes a preassigned value, whereby all complex values are to b considered. This change in method has also led to new insights in anothe direction: it has helped to bring the algebraic-analytic points of view basic to the Cauchy-Weierstrass function theory into closer contact wit the principles of the Riemannian function theory, which in greate measure emphasizes the potential theoretical and geometrical features o the analytic mappings under study.

This new edition contains some changes and additions, particularly concerning the Second Main Theorem. At the suggestion of severa colleagues, I have included my "elementary" proof of the theorem. also give a version of F. NEVANLINNA's differential geometrical methoc which makes the main theorem easier of access.

In the course of the last three decades the literature on the topic treated here and related questions has grown enormously. Of the work which contain important contributions to these problems I wish to mention in particular:

WEYL, H.: Meromorphic Curves and Analytic Curves. Princeton: Princeton University Press 1943.

HAYMAN, W. K.: Meromorphic Functions. Oxford: Clarendon Press 1964.

WITTICH, H.: Neuere Untersuchungen über eindeutige analytische Funktionen. Berlin-Göttingen-Heidelberg: Springer 1955.

This book has been translated into English with great care and interest by Dr. P. EMIG. For this I express to him my sincere thanks. I am also indebted to Cand. Phil. T. HEIKKURINEN for her help in the reading and correction of the proofs. Finally, I should like to express my thanks to Springer-Verlag for their friendly cooperation in the production of this volume.

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