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Series Editors

Herbert Spohn

Large Scale Dynamics of Interacting Particles

With 19 Figures



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For Sian and Shanti

Preface

The core of the material on large scale dynamics of interacting particles grew out of courses I taught at the Katholieke Universiteit Leuven, Rutgers University, and the Ludwig-Maximilians-Universität München and out of lectures I gave at the workshop “Hydrodynamical Behavior of Microscopic Systems” at the Università dell’Aquila.

I had the good luck of being helped through difficult ground by many friends. Amongst them I am deeply indebted to Joel L. Lebowitz. He got me started. Relatively little would have been achieved without his never-ending curiosity and insistence on clarity. Furthermore, I gratefully acknowledge the cooperation of Michael Aizenman, Henk van Beijeren, Carlo Boldrighini, Jean Bricmont, Paola Calderoni, Brian Davies, Anna DeMasi, Roland Dobrushin, Detlef Dürr, Gregory Eyink, Mark Fannes, Pablo Ferrari, Alberto Frigerio, Joseph Fritz, Antonio Galves, Shelly Goldstein, Vittorio Gorini, Reinhard Illner, Claude Kipnis, Joachim Krug, Oscar Lanford, Reinhard Lang, Joel Lebowitz, Christian Maes, Stefano Olla, George Papanicolaou, Errico Presutti, Mario Pulvirenti, Fraydoun Rezakhanlou, Hermann Rost, Yasha Sinai, Yuri Suhov, Domo Szász, Ragu Varadhan, André Verbeure, David Wick, and Horng-Tzer Yau. The list is somewhat lengthy, perhaps, but besides thanks I want to make clear that what I will describe is the outcome of a *common* scientific enterprise.

I thank Henk van Beijeren and Detlef Dürr for careful reading of and comments on a previous version. Paola Calderoni and Detlef Dürr supplied me with the proof in Part I, Chapter 8.4 which is most appreciated.

München, May 1991

Herbert Spohn

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