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Quantum Mechanics II

Translated by L. Alvarez-Gaumé

With 70 Figures

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Preface

The first edition of this book was published in 1978 and a new Spanish edition in 1989. When the first edition appeared, Professor A. Martin suggested that an English translation would meet with interest. Together with Professor A.S. Wightman, he tried to convince an American publisher to translate the book. Financial problems made this impossible. Later on, Professors E.H. Lieb and W. Thirring proposed to entrust Springer-Verlag with the translation of our book, and Professor W. Beiglböck accepted the plan. We are deeply grateful to all of them, since without their interest and enthusiasm this book would not have been translated.

In the twelve years that have passed since the first edition was published, beautiful experiments confirming some of the basic principles of quantum mechanics have been carried out, and the theory has been enriched with new, important developments. Due reference to all of this has been paid in this English edition, which implies that modifications have been made to several parts of the book. Instances of these modifications are, on the one hand, the neutron interferometry experiments on wave-particle duality and the 2π rotation for fermions, and the crucial experiments of Aspect et al. with laser technology on Bell's inequalities, and, on the other hand, some recent results on level ordering in central potentials, new techniques in the analysis of anharmonic oscillators, and perturbative expansions for the Stark and Zeeman effects.

Major changes have been introduced in presenting the path-integral formalism, owing to its increasing importance in the modern formulation of quantum field theory. Also, the existence of new and more rigorous results has led us to change our treatment of the W.B.K. method. A new section on the inverse scattering problem has been added, because of its relevance in quantum physics and in the theory of integrable systems. Finally, we have tried to repair some omissions in the first edition, such as lower bounds to the ground-state energy and perturbation expansions of higher order for two-electron atoms.

The material appears in a two-volume edition. Volume I contains Chaps. 1 to 7 and Appendices A to G. The physical foundations and basic principles of quantum mechanics are essentially covered in this part. States, observables, uncertainty relations, time evolution, quantum measurements, pictures, representations, path integrals, inverse scattering, angular momentum, symmetries, are among the topics dealt with. Simple dynamics under one-dimensional and central potentials provide useful illustrations. Pertinent mathematical complements are included in the Appendices as well as a brief introduction to some polemical issues as the collapse of quantum states and hidden variables. Volume II comprises Chaps. 8 to 15. Collision theory, W.B.K. approximation, stationary perturbation theory and variational method, time-dependent perturbations, identical particles, and the quantum theory of radiation are presented. As applications the Stark and Zeeman effects, the atomic fine structure, the van der Waals forces, the BCS theory of superconductivity and the interaction of radiation with matter are discussed.

To our acknowledgements we wish to add a special one to G. García Alcaine for providing us with highly valuable bibliographic material for the updating of this book. We also thank Professor L. Alvarez-Gaumé for the interest he showed in translating the manuscript of the second Spanish edition. We are delighted with his excellent work. Finally, we would like to thank Springer-Verlag for all their help.

Madrid, May 1990 *A. Galindo, P. Pascual*

Preface to the Spanish Edition

Quantum mechanics is one of the basic pillars of physics. It is therefore not surprising that new results have been continuously accumulating, not only in a wide variety of applications, but also in the very foundations, which were laid down almost entirely in the spectacular burst of creative activity of the 1920s. Above all else, these efforts were, and are, intended to establish its axiomatic scheme and to make more rigorous many of the methods commonly employed.

Following the classic texts by Dirac and von Neumann, a large number of books on quantum mechanics and its mathematical and physical foundations have appeared, many of which are excellent. Our reasons for adding another one to the list arise from long teaching experience in the subject. First was the necessity of collecting in one book the clarification of many points which, in our opinion, remain obscure in traditional treatments. Second was the desire to incorporate important material which in general can only be found dispersed among scientific journals or in specialized monographs.

Given the huge quantity of information available, we decided to confine ourselves to what is normally termed "nonrelativistic quantum mechanics". The omission of the relativistic aspects of physical systems is due to our conviction that, for reasons of physics rather than textbook space, developments of relativistic quantum theory should be more than simply the last chapter in a textbook on quantum mechanics. Additionally, Chaps. 1 and 14 were included since it seemed appropriate to include a historical perspective of the genesis of quantum mechanics and its physical basis (albeit a short one since our physics majors have already had a course in quantum physics). Also, given that the atom plays a central role in quantum mechanics, we decided to use it to illustrate (Chap. 14) how one deals with a complex system using the approximation techniques developed in previous chapters. In addition to restricting the scope of the book, we sometimes found it necessary (with the length of the work in mind) to summarize or omit altogether some points of unquestionable interest, e.g., in Chap. 8 the study of dispersion relations, Regge poles, the Glauber approximation, etc.

A major problem which presented itself at the outset was that of choosing the mathematical level of the work. Aiming essentially at students of physics and related sciences, we adopted as a base the knowledge which these students acquire in normal undergraduate training. Thus, we started with the supposition of a certain familiarity with the language and at least the elementary techniques of Hilbert spaces, groups, etc., although at times our desire to make concepts more precise or to include (generally without proof) rigorous results have forced us to use more advanced mathematical terminology. With the aim of helping the reader who wishes to pursue such aspects, in Appendix C we summarize the relevant points and provide a pertinent bibliography.

The postulational basis of quantum mechanics is developed in Chap. 2 in the spirit of the orthodox "Copenhagen" interpretation, and the content of (and necessity for) each postulate is discussed. This approach is completed in Chap. 13 with the symmetrization principle for systems of identical particles. Other more controversial aspects, such as the problem of measurement and hidden variable theory, are discussed briefly in Appendices E and F.

In Chaps. 3 to 8 we develop the quantum principles, introducing some basic concepts with applications to simple systems, the majority of which are exactly soluble. Chapter 3 is devoted to the study of the wave function and its time evolution, and ends with a short introduction to Feynman's alternative formulation of quantum mechanics.

The properties of bound states in essentially one-dimensional systems are treated in Chaps. 4 and 6, the collision problem being included in Chap. 4 (scattering states for one-dimensional problems) and comprising all of Chap. 8. In Chap. 5 angular momentum is studied in detail and, given its practical importance, this study is completed in Appendix B with a summary of the most common formulae and tables of Clebsch-Gordan and Racah coefficients. Finally, Chap. 7 contains a discussion of symmetry transformations, the most important invariances of physical systems, and the associated conservation laws.

The dynamical complexity of the majority of interesting physical problems makes it necessary to resort to approximation methods in order to understand their behaviour. In Chaps. 9 and 10 the techniques appropriate to a discussion of stationary states are presented, reserving for Chap. 11 the usual methods for time-dependent perturbations. In Chap. 12 we study charged particles moving in electromagnetic fields, discuss the gauge-invariance of their dynamics and apply the above-mentioned perturbative methods to calculate the fine structure and the Zeeman effect in hydrogenic atoms.

The symmetrization principle is developed in Chap. 13, introducing second quantization formalism and applying it to many-body systems of identical particles displaying quantum behaviour at the macroscopic level (the phenomena of superfluidity and superconductivity). In Chap. 15, the last one of the book, we present a treatment, relatively complete, of the problem of the interaction of radiation with matter, with the necessary quantization of the radiation field.

The book concludes with a collection of appendices which, in addition to those mentioned above, include a summary of the most important special functions (Appendix A), elements of the theory of distributions and Fourier transforms (Appendix D), and properties of certain antiunitary operators (Appendix G). Contrary to the usual custom, we have not included problems or exercises. Our intent is to collect these in a forthcoming book, which will complement the present work in a practical sense.

The mathematical notation utilized in this work is the traditional one, although in some instances, to simplify the formulae, we have omitted symbols where there can be little room for confusion. Thus, for example, we use indistinguishably $\lambda I - A$ and $\lambda - A$. We also omit the limits or domains of integration on many definite integrals when they coincide with the natural ones. Concerning the numbering of equations, (3.37) or (X.37) indicates equation number 37 of Chap. 3 or Appendix X, respectively. Finally, [XY 57] is a reference to the article or work by a single author whose last name begins with XY, or of various authors whose last names start with X and Y, respectively; the number indicates the last two digits of the year in which the publication appeared. When necessary, we use [XY 57n] to distinguish between analogous cases.

To conclude, we wish to express, first and foremost, our gratitude to our families, whose understanding, sacrifice and support have permitted us to dedicate many hours to the realization of this book. We also wish to show our gratitude to various coworkers for their critical reviews of various chapters and appendices: L. Abellanas (Appendices C and D); R. F. Alvarez-Estrada (Chap. 8); R. Guardiola (Chap. 15); A. Morales (Chap. 7); A. F. Rañada (Chaps. 2 and 3); and C. Sánchez del Rio (Chaps. 1 and 14). Their suggestions were extremely valuable. G. García-Alcaine and M. A. Goñi read considerable portions of the original, and we also benefitted from their comments. Lastly, we appreciate Sra. C. Marcos' typing of a first draft of this work, and especially the patience and care with which Srta. M. A. Iglesias typed the text and formulae of the final version.

We do not wish to end this prologue without thanking the editorial staff of Alhambra, who were most cooperative in accepting our suggestions during the preparation of this book.

Contents

Physical Constants

Avogadro number	$N_A = 6.022\,045\,(31) \times 10^{23}$ particles/mol
Speed of light	$c = 2.997\,924\,58\,(1.2) \times 10^{10}$ cm s^{-1}
Proton charge	$\lvert e \rvert = 4.803\,242\,(14) \times 10^{-10}$ Fr $= 1.602\,189\,2\,(46) \times 10^{-19}$ C
Reduced Planck constant	$\hbar = 6.582\,173\,(17) \times 10^{-22}$ MeV s $= 1.054\,588\,7\,(57) \times 10^{-27}$ erg s $\hbar c = 1.973\,285\,8\,(51) \times 10^{-11}$ MeV cm $= 197.328\,58\,(51)$ MeV fm $= 0.624\,007\,8\,(16)$ GeV mb$^{1/2}$
Fine structure constant	$\alpha = e^2/\hbar c = 1/137.036\,04\,(11)$
Boltzmann constant	$k_B = 1.380\,662\,(44) \times 10^{-16}$ erg K^{-1} $= 8.617\,35\,(28) \times 10^{-11}$ MeV K^{-1}
Electron mass	$m_e = 9.109\,534\,(47) \times 10^{-28}$ g $m_e c^2 = 0.511\,003\,4\,(14)$ MeV
Proton mass	$m_p = 1836.151\,52\,(70)\, m_e$ $m_p c^2 = 938.279\,6\,(27)$ MeV
Electron Compton wavelength	$\lambda_e = h/m_e c = 2.426\,308\,9\,(40) \times 10^{-10}$ cm
Bohr radius	$a_{\infty \mathrm{Bohr}} = 0.529\,177\,06\,(44)$ Å
Rydberg (energy)	$R_\infty = \frac{1}{2} m_e (\alpha c)^2 = 13.605\,804\,(36)$ eV
Rydberg (frequency)	$R_\infty = m_e (\alpha c)^2 / 4\pi\hbar = 3.289\,842\,(17) \times 10^{15}$ Hz
Rydberg (wave number)	$R_\infty = m_e (\alpha c)^2 / 4\pi\hbar c = 109\,737.32\,(56)$ cm^{-1}
Bohr magneton	$\mu_B = \hbar \lvert e \rvert / 2 m_e c = 0.578\,837\,85\,(95) \times 10^{-14}$ MeV G^{-1}
Nuclear magneton	$\mu_N = \hbar \lvert e \rvert / 2 m_p c = 3.152\,451\,5\,(53) \times 10^{-18}$ MeV G^{-1}
Gravitational constant	$G = 6.672\,0\,(41) \times 10^{-8}$ cm^3 g^{-1} s^{-2}
Gravitational fine structure constant	$\alpha_G = \dfrac{G m_p^2}{\hbar c} = 5.904\,2\,(36) \times 10^{-39}$

Note: The values of these constants have been taken from [AC 84]. Numbers within brackets correspond to one standard deviation and affect the last digits. It should be remembered that in October 1983 the General Conference on Weights