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Introduction to Gauge Field Theories

With 75 Figures



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Preface

In recent years, gauge fields have attracted much attention in elementary particle physics. The reason is that great progress has been achieved in solving a number of important problems of field theory and elementary particle physics by means of the quantum theory of gauge fields. This refers, in particular, to constructing unified gauge models and theory of strong interactions between the elementary particles.

This book expounds the fundamentals of the quantum theory of gauge fields and its application for constructing unified gauge models and the theory of strong interactions.

In writing the book, the authors' aim was three-fold: firstly, to outline the basic ideas underlying the unified gauge models and the theory of strong interactions; secondly, to discuss the major unified gauge models, the theory of strong interactions and their experimental implications; and, thirdly, to acquaint the reader with a rather special mathematical approach (path-integral method) which has proved to be well suited for constructing the quantum theory of gauge fields.

Gauge fields are a vigorously developing area. In this book, we have selected for presentation the more or less traditional and commonly accepted material. There also exist a number of different approaches which are presently being developed. The most important of them are touched upon in the Conclusion.

The book is intended for those who are familiar with the basic facts of elementary particle physics and with the fundamentals of relativistic quantum mechanics. In order to enable the reader to avoid the necessity of consulting different sources, we present in Chap. 1 the basic facts about the global groups of the space-time and the internal symmetry, about the Lagrangians invariant under these groups as well as about the operator form of the quantum field theory.

During the preparation of this book the authors have benefited from discussing various questions with many of their colleagues. It is a pleasure to express gratitude to all of them and to acknowledge the stimulating discussions and their useful advice.

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Helsinki, Moscow, February 1984

M. Chaichian, N. F. Nelipa

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