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Henri J. Nussbaumer

Fast Fourier Transform and Convolution Algorithms

Second Corrected and Updated Edition

With 38 Figures

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Preface to the Second Edition

In the first edition of this book, we covered in Chapter 6 and 7 the applications to multidimensional convolutions and DFT's of the transforms which we have introduced, back in 1977, and called polynomial transforms. Since the publication of the first edition of this book, several important new developments concerning the polynomial transforms have taken place, and we have included, in this edition, a discussion of the relationship between DFT and convolution polynomial transform algorithms. This material is covered in Appendix A, along with a presentation of new convolution polynomial transform algorithms and with the application of polynomial transforms to the computation of multidimensional cosine transforms.

We have found that the short convolution and polynomial product algorithms of Chap. 3 have been used extensively. This prompted us to include, in this edition, several new one-dimensional and two-dimensional polynomial product algorithms which are listed in Appendix B.

Since our book is being used as part of several graduate-level courses taught at various universities, we have added, to this edition, a set of problems which cover Chaps. 2 to 8. Some of these problems serve also to illustrate some research work on DFT and convolution algorithms.

I am indebted to Mrs A. Schlageter who prepared the manuscript of this second edition.

Lausanne
April 1982

HENRI J. NUSSBAUMER

Preface to the First Edition

This book presents in a unified way the various fast algorithms that are used for the implementation of digital filters and the evaluation of discrete Fourier transforms.

The book consists of eight chapters. The first two chapters are devoted to background information and to introductory material on number theory and polynomial algebra. This section is limited to the basic concepts as they apply to other parts of the book. Thus, we have restricted our discussion of number theory to congruences, primitive roots, quadratic residues, and to the properties of Mersenne and Fermat numbers. The section on polynomial algebra deals primarily with the divisibility and congruence properties of polynomials and with algebraic computational complexity.

The rest of the book is focused directly on fast digital filtering and discrete Fourier transform algorithms. We have attempted to present these techniques in a unified way by using polynomial algebra as extensively as possible. This objective has led us to reformulate many of the algorithms which are discussed in the book. It has been our experience that such a presentation serves to clarify the relationship between the algorithms and often provides clues to improved computation techniques.

Chapter 3 reviews the fast digital filtering algorithms, with emphasis on algebraic methods and on the evaluation of one-dimensional circular convolutions.

Chapters 4 and 5 present the fast Fourier transform and the Winograd Fourier transform algorithm.

We introduce in Chaps. 6 and 7 the concept polynomial transforms and we show that these transforms are an important tool for the understanding of the structure of multidimensional convolutions and discrete Fourier transforms and for the design of improved algorithms. In Chap. 8, we extend these concepts to the computation of one-dimensional convolutions by replacing finite fields of polynomials by finite fields of numbers. This facilitates introduction of number theoretic transforms which are useful for the fast computation of convolutions via modular arithmetic.

Convolutions and discrete Fourier transforms have many uses in physics and it is our hope that this book will prompt some additional research in these areas and will help potential users to evaluate and apply these techniques. We also feel that some of the methods presented here are quite general and might someday find new unexpected applications.

Part of the material presented here has evolved from a graduate-level course taught at the University of Nice, France. I would like to express my thanks to Dr. T.A. Kriz from IBM FSD for kindly reviewing the manuscript and for making many useful suggestions. I am grateful to Mr. P. Bellot, IBM, C.E.R., La Gaude, France, for his advice concerning the introductory chapter on number theory and polynomial algebra, and to Dr. J.W. Cooley, from IBM Research, Yorktown Heights, for his comments on some of the work which led to this book. Thanks are also due to Dr. P. Quandalle who worked with me on polynomial transforms while preparing his doctorate degree and with whom I had many fruitful discussions. I am indebted to Mrs. C. De Backer for her aid in improving the English and to Mrs. C. Chevalier who prepared the manuscript.

La Gaude
November 1980

HENRI J. NUSSBAUMER

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