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Inverse Scattering Problems in Optics

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With a Foreword by R. Jost

With 49 Figures

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The Integral Equation for Incomplete Information (for the Physical Optics Inverse Scattering Identity), and the High Frequency Solution (to this integral equation) utilizing the directional derivative (Bojarski, N. N., "Electromagnetic Inverse Scattering Theory", Syracuse University Research Corporation, Special Projects Laboratory Rpt. SFL TR 68-70, AD 509134, December 1968). The first numerico-experimental verification of the high frequency solution was reported in "Electromagnetic Inverse Scattering" (Bojarski, N. N., Naval Air Systems Command Report, June 1972)."

Foreword

When, in the spring of 1979, H.P. Baltes presented me with the precursor of this volume, the book on "Inverse Source Problems in Optics", I expressed my gratitude in a short note, which in translation, reads:

"Dear Dr. Baltes, the mere title of your unexpected gift evokes memories of a period, which, in the terminology of your own contribution, would be described as the Stone Age of the Inverse Problem. Those were pleasant times. Walter Kohn and I lived in a cave by ourselves, drew pictures on the walls, and nobody seemed to care. Now, however, Inversion has become an Industry, which I contemplate with as much bewilderment as a surviving Tasmanian aborigine gazing at a modern oil refinery with its towers, its flares, and the confusing maze of its tubes."

The present volume makes me feel even more aboriginal – impossible for me to fathom its content. What I can point out, however, is one of the forgotten origins of the Inverse Scattering Problem of Quantum Mechanics: Werner Heisenberg's "S-Matrix Theory" of 1943. This grandiose scheme had the purpose of eliminating the notion of the Hamiltonian in favour of the scattering operator. If Successful, it would have done away once and for all with any kind of inverse problem.

To my knowledge, it was Wolfgang Pauli who induced S.T. Ma to check Heisenberg's idea in simple nonrelativistic models. Was it really possible to deduce all observable quantities of a system of two particles from its scattering data? Was Heisenberg's prescription for the determination of bound states infallible? S.T. Ma found a counterexample, where the S matrix had zeros which were uncorrelated with bound states. I learnt about Ma's work from Ch. Møller in Copenhagen and it finally led me to the inverse scattering problem. The reader may find more about the continuation of this story in the Bargmann Festschrift *Studies in Mathematical Physics*, ed. by E.H. Lieb et al. (Princeton University Press 1976).

ETH Zürich, November 1979

Res Jost

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