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Mathematical Demography

Selected Papers

With 31 Figures

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For John and Harriet

Preface

This volume is an effort to bring together important contributions to the mathematical development of demography and to suggest briefly their historical context. We have tried to find who first thought of the several concepts and devices commonly used by demographers, what sort of problem he was facing to which the device or concept seemed the solution, and how his invention developed subsequently in the hands of others.

Historically, the book starts with a Roman table of life expectancies from the third century a.d. about which we know little, and with John Graunt's explorations in an area that was still popularly suspect when he wrote in 1662. These are followed by the astronomer Halley, who looked into the field long enough to invent the life table and to notice that Their Majesties would take a sizeable loss on the annuity scheme they had just launched; and by Euler, who was first to devise the formulas of stable population theory and to apply them to filling gaps in data. To these we add the handful of further contributions in the 19th century and many pieces from the explosion of contributions that began in this century with Lotka. We doubt that we have managed to trace everything back to its ultimate beginning, and suspect that our nominees in some cases have been anticipated by predecessors who will be turned up by other students.

The works we include form a living heritage in demography: Graunt; Halley; Euler; Lotka; Milne, who formalized life table construction; Lexis, who was preoccupied with the way members of a population are situated simultaneously in age and in time, and showed how a plane chart, now known as a Lexis diagram, can help analysis. Much less alive, and largely excluded here, are such notions as that of George King, that graduation of data for a life table was more accurate from pivotal death rates calculated at five-year intervals; John Graunt's belief that the right way to describe the dynamics of a population was as the *ratio* of births to deaths, without considering age; and devices that once reduced the labor of numerical calculation but are obsolete in a computer age. These and many other ideas that have proved to be dead ends and are now of merely antiquarian interest we tried to distinguish from those that were part of a chain of development that is still advancing. As far as we could discriminate our excerpts are confined to the latter.

To determine which works most deserve attention among the large number written has not been easy, and we have undoubtedly made mistakes both of inclusion and of omission. We were far from insisting on subtle mathematical ideas, but did look for the effective uses of mathematics that have come to be assimilated into population work. Articles that profess to deal with population but whose main interest was mathematics we tried to avoid, and we avoided them doubly if they were a mere import from some other subject that seemed unlikely ever to be naturalized in population analysis. Some ideas and techniques have a kind of *droit de la cité* in contemporary population study, and we hope these are the ones that predominate in our selections.

To find passages that were self-contained and suitable for contemporary reading was occasionally difficult. Writers often used symbols well known to their place and time, and their immediate readership had no need for definitions we would now miss. To this the earlier works add key formulas with no hint as to how they are derived. Where we expect readers to have trouble as a result, because we did, we include a brief explanation of what is being done.

The choice of excerpts from the classic articles and books rather than complete reprints in all cases was dictated partly by economy of publication, but this was not the only constraint. Benjamin Gompertz fairly compactly introduces his Law of mortality, but spends above fifty pages fitting it to life tables and working out its implications for annuity payments. Harro Bernardelli published the first article on the use of matrices in population projection in the *Journal of the Burma Research Society*, which is not a source that most of us would come across in our ordinary reading. He has top priority for inclusion, but he deals partly with problems of the Burmese economy under British colonial rule and with speculations on cyclic events that do not carry much interest for readers today. Leslie, whose reading in a sickbed had taken him deep into the mathematical properties of matrices, went into cogredient and contragredient transformations that are unlikely to have demographic application. We saw no need to burden the reader with these only to have him discover at the end that he would never need them.

In editing we did not strip down our authors to the point of losing the context of their contribution to our subject. We learned much of an incidental character in our reading and have tried to retain that richness. Where substantive omissions are made we note these for the reader's benefit.

Several topics that fall in the province of demography are not included, among them treatments of human spatial ecology, urbanization, and migration. Omission is partly due to space limitations, and partly to lack of confidence in our ability to decide what is basic in fields whose mathematical explorations are recent and expanding rapidly.

We expect from the reader at least some background in calculus and matrix algebra, and several papers will require an understanding of stochastic processes. The reader lacking a background in elementary mathematics will find the greater part of the book difficult.

Secondary accounts of much of what we present can be found in Keyfitz (1968), and stochastic processes are well handled in Feller (1968) and Chiang (1968). Our chief sources for the early histories given here are Hendriks (1852, 1853), Westergaard (1969), and Lorimer (1959).

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