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E. Schöll

Nonequilibrium Phase Transitions in Semiconductors

Self-Organization Induced by Generation and Recombination Processes

With 165 Figures

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To my parents

Preface

Semiconductors are complex dynamic systems which can in many cases exhibit electrical instabilities like current runaway, switching between a nonconducting and a conducting state, or spontaneous oscillations of current or voltage when they are driven far from thermodynamic equilibrium by strong external electric fields, irradiation, or current injection. There are a variety of different physical mechanisms that can give rise to such instabilities, but the observed phenomena are often similar, involving the spontaneous formation of spatial and temporal structures. Such *self-organizing* cooperative processes have been noted in a great number of different physical, chemical, and biological "synergetic" systems, when a state *far from thermodynamic equilibrium* is maintained by a continuous flux of energy or matter flowing through them. These instabilities bear a remarkable analogy with *phase transitions* of systems in thermal equilibrium, like ferromagnets or real gases.

Self-organization and nonequilibrium phase transitions in semiconductors are of considerable current interest for at least two reasons. First, these phenomena are the basis of a number of important semiconductor devices which are used in modern microelectronics and semiconductor technology. Second, semiconductors are particularly apt model systems for the study of complex nonlinear dynamics and self-organization, since recent advances in the technology of tailoring specific samples ("materials engineering"), and the direct observation via current and voltage measurements allowing for good reproducibility and high temporal and spatial resolution have opened up the possibility of a fruitful interaction between theory and experiment.

In this book we concentrate on those semiconductor instabilities whose physical mechanism is based upon nonlinear generation and recombination processes of the charge carriers. The aim of the book is to provide a coherent theoretical description of the spatial and temporal structures induced by simple generation-recombination mechanisms. It attempts to go beyond older work by taking into account in a systematic way the *nonlinearities* of these generation-recombination processes and all dynamic degrees of freedom of free *and trapped* carriers, and by introducing and developing the concepts of nonequilibrium phase transitions. A comparison with experiments is drawn where appropriate. The emphasis is on recent theoretical and experimental advances in this still-open field, and no comprehensive historic overview is attempted.

This work is intended to connect the field of semiconductor physics and the theory of nonlinear dynamic systems far from thermodynamic equilibrium. It might therefore be useful both to applied semiconductor physicists and to theoreticians by providing, on one hand, new concepts and viewpoints for the understanding of certain semiconductor instabilities, and, on the other hand, by presenting a new application of the theory of synergetic systems. The organization of the material is as follows. Chapter 1 contains the basic physical principles and some background of semiconductor instabilities and nonequilibrium phase transitions. In Chap. 2 a number of simple generation-recombination mechanisms for nonequilibrium phase transitions are presented, and potential applications to threshold switching in crystalline and amorphous semiconductors, nonlinear magneto-photoconductivity, and optical bistability are discussed. In Chaps. 3–6 the spatial and temporal structures resulting from the primary instability are developed. Chapter 3 discusses the linear modes describing the initial stages of the spatial and temporal instabilities. In Chap. 4 the fully developed stationary spatial structures (current filaments) are analyzed. Their stability, and transient phenomena like spinodal decomposition and nucleation processes are discussed in Chap. 5. Chapter 6 deals with temporal and spatio-temporal structures, in particular mechanisms for oscillations and for chaos are developed, and applied to explain recent experiments, in which intrinsic chaos was observed in semiconductors under a variety of experimental conditions.

I am very much indebted to Prof. P.T. Landsberg, Southampton/Gainesville, Fla., and Prof. F. Schlögl, Aachen, who have greatly influenced and stimulated the development of the ideas presented in this book. In the past I have benefited from valuable discussions with Prof. D. Bimberg, Berlin; Prof. V. Dohm, Aachen; Prof. B.C. Eu, Montreal; Dr. H.L. Grubin, Glastonbury, Conn.; Prof. R.P. Huebener, Tübingen; Prof. N. Klein, Haifa; Dr. V.V. Mitin, Kiev; Dr. J. Parisi and J. Peinke, Tübingen; Prof. W. Prettl, Regensburg; Prof. J. Schnakenberg, Aachen; Prof. M.P. Shaw, Detroit, Mich.; Prof. A. Stahl, Aachen; Prof. H. Thomas, Basel, and my colleagues at the Institute of Theoretical Physics of the RWTH Aachen, in particular Drs. C. Escher and W. Renz. I am obliged to Mrs. B. Schumacher for checking some of the formulas.

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Finally, I wish to offer sincere thanks to my wife and my children for their patience and encouragement during all stages of writing.

Aachen, April 1987 *Eckehard Schöll*

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