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Fred Kröger

Temporal Logic of Programs

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Preface

Temporal logic is a logic of propositions whose truth and falsity may depend on time. Closely related to modal logics, it has been studied for a long time. Precise formal foundations of (various kinds of) temporal logic have been laid during the last, say, 25 years.

In classical mathematics propositions do not depend on time (they are *static* in some sense), so temporal logic is not of much interest there. The mathematical treatment of *programs*, however, contains a significant *dynamic* aspect. A typical model of the execution of a program is a sequence of *states* (in the “flow of time”). In different states, program entities such as variables may have different values and, hence, propositions about these values may have different truth values. *Temporal logic of programs* means taking temporal logic as a basis for the mathematics of execution sequences of programs and applying the logical means – language and deduction mechanisms – to the formal description and analysis of dynamic program properties.

From its very beginning about 10 years ago, this approach has received much attention and been a remarkable success, in particular in the field of parallel programs. It still constitutes a large area of present-day research. This monograph – an elaboration of the notes from courses given in the winter semesters 1983/84 and 1984/85 at the Technical University of Munich – tries to give a comprehensive and uniform presentation of some of the material which has been developed during the last few years and has now apparently reached some sort of “saturated state”. Moreover, putting together the various notions, methods, results and applications we have paid much attention to precisely elaborated proofs and intuitive motivations and explanations of technical details. So the book may also serve as a textbook for graduate students and teachers.

The content of the book is divided into three main parts. Chapters I–III deal with the pure (“linear time”) temporal logic. In Chapter I the new linguistic features with their formal semantics are introduced. In Chapter II the new (propositional) logic is axiomatized, and in Chapter III it is extended to a first-order logic.

Chapter IV describes the “temporal semantics” of (parallel) programs, i.e., how to represent programs and their properties within the language of temporal logic.

Chapters V–VII present applications of the logical apparatus to the verification of program properties. Chapter V deals with *invariance* and *precedence* properties; *liveness* properties are treated in Chapter VI. The development of the basic proof methods in Sections 16, 18, 21 and 22 is accompanied by various elaborated examples for their application in Sections 17, 19 and 23. A discussion of special methods for sequential programs in Chapter VII concludes this third part.

The reader of this book is assumed to be familiar with the general concepts of mathematical logic and the main concepts of classical propositional and first-order logic. In the Introduction we give only a short summary of those notions which will be needed in the following.

The book presents material from many sources. We have not included all the corresponding citations in the text, but in separate bibliographical remarks at the end of the book we have tried to give – hopefully – the proper credit to everyone whose publications have been used.

I am grateful to A. Kausche, K. Klaus, H. Schlingloff, F. Stolz and – in particular – H. Vogel who helped in debugging preliminary versions of this monograph and provided important suggestions for several improvements.

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Munich, November 1986

Fred Kröger

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