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Chaos and Statistical Methods

Proceedings of the Sixth Kyoto Summer Institute,
Kyoto, Japan
September 12 - 15, 1983

Editor: Y. Kuramoto

With 123 Figures

Springer-Verlag
Berlin Heidelberg New York Tokyo 1984

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Series Editor:

Professor Dr. Dr. h. c. Hermann Haken

Institut für Theoretische Physik der Universität Stuttgart, Pfaffenwaldring 57/IV,
D-7000 Stuttgart 80, Fed. Rep. of Germany

ISBN-13: 978-3-642-69561-2 e-ISBN-13: 978-3-642-69559-9
DOI: 10.1007/978-3-642-69559-9

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Softcover reprint of the hardcover 1st edition 1984

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2153/3130-543210

Preface

The 6th Kyoto Summer Institute devoted to "Chaos and Statistical Mechanics" was held from September 12 to 15, 1983, at the Research Institute for Mathematical Sciences, Kyoto University, and at Hotel Kuniso. The meeting was aimed at clarifying various aspects of chaotic systems appearing in different scientific disciplines, critically examining related mathematical methods developed so far, thus preparing for possible breakthroughs, among others, for the opening of a new period of statistical mechanics of deterministic systems. The number of participants was 135, of which 24 were from abroad. We believe that the well-prepared lecture of each speaker and lively discussions among many participants from various research fields led the meeting to a successful conclusion.

The 6th KSI was organized by the Research Institute for Fundamental Physics. A number of young chaos researchers in Japan also participated actively in the organization. We were also in close contact with the organizer of the IUTAM Symposium on "Turbulence and Chaotic Phenomena in Fluids" (Kyoto Kaikan Conference Hall, Kyoto, September 5-10 1983).

This volume contains most of the lectures presented at the 6th KSI. We are very grateful to all the authors for their efforts in preparing such excellent manuscripts.

The 6th KSI was supported by the Ministry of Education, Science and Culture and the Yamada Science Foundation. The organizing committee acknowledges gratefully their generous financial support. Finally, thanks are due to Dr. M. Toya and Miss T. Sumide for their invaluable assistance.

Kyoto, January, 1984

Yoshiki Kuramoto

Opening Address

Ladies and gentlemen,

It is a great pleasure and privilege for me to present the opening address at this 6th Kyoto Summer Institute on Chaos and Statistical Mechanics. On behalf of the Research Institute for Fundamental Physics, Kyoto University, I am delighted to welcome all of you to this international meeting. And I would like to thank all participants, especially those who have traveled far distances to contribute to the meeting. I wish everyone a pleasant stay in Kyoto.

The series of Kyoto Summer Institute, called in short "KSI", started in 1978, and ever since it has been held every year. The topics of KSI in the past varied from year to year, and they ranged from particle physics to condensed matter physics. They were all carefully selected topics so that they could cover most important and active research areas of fundamental physics. This year we selected a topic on chaos. I believe this timely subject is particularly suited to the spirit of KSI. In fact, everyone knows the marvelous progress achieved recently in this area both in theory and experiment. Moreover, the notion of chaos is becoming far more familiar than in the past in various fields of science. This seems to be true not only for such fields as fluid dynamics and statistical mechanics, but also for quite different fields such as condensed matter physics, plasma physics, nuclear physics, chemical reactions and even some branches of biology.

There exists certainly added significance to this 6th KSI, which I would like to mention briefly. As is well known, the Research Institute for Fundamental Physics was founded by the late Dr. Hideki Yukawa, which was exactly 30 years ago. In this connection, we regard the present 6th KSI as one of the most important activities in the celebration program of the 30th anniversary of its foundation. It is gratifying that the 6th KSI has taken up an attractive field of highly interdisciplinary nature, because our institute has, since its foundation, always encouraged and contributed itself to such research fields that looked still premature but might possess great potentiality for future development.

Finally, I hope that this meeting will lead to better understanding of chaos and statistical mechanics through lively discussions among scientists from different parts of the world and from different scientific disciplines.

Before closing my address, I should like to thank the Ministry of Education, Science and Culture and the Yamada Science Foundation for their financial support, and the Research Institute for Mathematical Sciences for kindly providing us with such a nice lecture hall.

With these few words, I now wish to declare the 6th KSI open. Thank you.

Ziro Marki

Director of the
Research Institute
for Fundamental Physics

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