

Beniamino Segre

Some Properties
of Differentiable Varieties
and Transformations

With Special Reference
to the Analytic and Algebraic Cases

Second Edition

With an additional part
written in collaboration with
J. W. P. Hirschfeld



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Preface to the First Edition

The present volume contains, together with numerous addition and extensions, the course of lectures which I gave at Pavia (26 September till 5 October 1955) by invitation of the «Centro Internazionale Matematico Estivo». The treatment has the character of a monograph, and presents various novel features, both in form and in substance; these are indicated in the notes which will be found at the beginning and end of each chapter,

Of the nine parts into which the work is divided, the first four are essentially differential in character, the next three deal with algebraic geometry, while the last two are concerned with certain aspects of the theory of differential equations and of correspondences between topological varieties. A glance at the index will suffice to give a more exact idea of the range and variety of the contents, whose chief characteristic is that of establishing suggestive and sometimes unforeseen relations between apparently diverse subjects (e. g. differential geometry in the small and also in the large, algebraic geometry, function theory, topology, etc.); prominence is given throughout to the geometrical viewpoint, and tedious calculations are as far as possible avoided.

The exposition has been planned so that it can be followed without much difficulty even by readers who have no special knowledge of the subjects treated. Although, for reasons of space, various proofs are here only sketched, nevertheless the essence of the methods is always made clear, while, on the other hand, at the end of each chapter there are sufficient references to enable the reader to pursue matters further. However, in certain cases cognate results have been omitted for the sake of brevity; for the same reason, the Bibliography does not claim to be complete.

I have had the good fortune to have received valuable assistance from Dr. DAVID KIRBY in preparing the English version of the text, from Dr. EDOARDO VESENTINI in typing and revising the typescript, and from Prof. LEONARD ROTH in reading the proofs; to all these I tender hearty and grateful thanks. In conclusion, I wish to express to the Springer-Verlag my appreciation of the superlatively good printing and production of this book.

Preface to the Second Edition

This edition differs from the first mainly in the insertion of a new Part Nine entitled "Projective Differential Geometry of Systems of Linear Partial Differential Equations", thus filling a gap in the theory of differentiable varieties. Part Nine of the previous edition is now Part Ten. Various misprints and errors have also been corrected. Otherwise the first edition remains intact.

I am most grateful to Dr. JAMES HIRSCHFELD for his valuable assistance in the compilation of the new Part and for his assiduity in tracing the misprints.

Roma, 29. IX. 1970

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