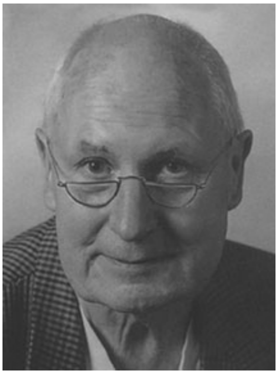

J. W. S. Cassels (known to his friends by the Gaelic form "Ian" of his first name) was born of mixed English-Scottish parentage on 11 July 1922 in the picturesque cathedral city of Durham. With a first degree from Edinburgh, he commenced research in Cambridge in 1946 under L. J. Mordell, who had just succeeded G. H. Hardy in the Sadleirian Chair of Pure Mathematics. He obtained his doctorate and was elected a Fellow of Trinity College in 1949. After a year in Manchester, he returned to Cambridge and in 1967 became Sadleirian Professor. He was Head of the Department of Pure Mathematics and Mathematical Statistics from 1969 until he retired in 1984.

Cassels has contributed to several areas of number theory and written a number of other expository books:

- *An introduction to diophantine approximations*
- *Rational quadratic forms*
- *Economics for mathematicians*
- *Local fields*
- *Lectures on elliptic curves*
- *Prolegomena to a middlebrow arithmetic of curves of genus 2* (with E. V. Flynn).

Classics in Mathematics

J.W.S. Cassels An Introduction to the Geometry of Numbers

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J.W.S. Cassels

An Introduction to the Geometry of Numbers

Reprint of the 1971 Edition

Springer

J.W.S. Cassels
University of Cambridge
Department of Pure Mathematics
and Mathematical Statistics
16, Mill Lane
CB2 1SB Cambridge
United Kingdom

Originally published as Vol. 99 of the
Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen

Mathematics Subject Classification (1991): 10Exx

CIP data applied for

Die Deutsche Bibliothek - CIP-Einheitsaufnahme

Cassels, John W.S.:
An introduction to the geometry of numbers / J.W.S. Cassels - Reprint of the 1971 ed. - Berlin; Heidelberg; New York; Barcelona; Budapest; Hong Kong; London; Milan; Paris; Santa Clara; Singapore; Tokyo: Springer, 1997
(Classics in mathematics)
ISBN-13: 978-3-540-61788-4 e-ISBN-13: 978-3-642-62035-5
DOI: 10.1007/978-3-642-62035-5

SPIN 10554506 41/3143-5 4 3 2 1 0 – Printed on acid-free paper

J. W. S. Cassels

An Introduction to the Geometry of Numbers

Second Printing, Corrected

Springer-Verlag Berlin · Heidelberg · New York 1971

Prof. Dr. J. W. S. Cassels
Professor of Mathematics, University of Cambridge, G. B.

AMS Subject Classifications (1970): 10 E xx

Preface

> Of making many bookes there is no end, and much studie is a wearinesse of the flesh.
>
> *Ecclesiastes* XII, 12.

When I first took an interest in the Geometry of Numbers, I was struck by the absence of any book which gave the essential skeleton of the subject as it was known to the experienced workers in the subject. Since then the subject has developed, as will be clear from the dates of the papers cited in the bibliography, but the need for a book remains. This is an attempt to fill the gap. It aspires to acquaint the reader with the main lines of development, so that he may with ease and pleasure follow up the things which interest him in the periodical literature. I have attempted to make the account as self-contained as possible.

References are usually given to the more recent papers dealing with a particular topic, or to those with a good bibliography. They are given only to enable the reader to amplify the account in the text and are not intended to give a historical picture. To give anything like a reasonable account of the history of the subject would have involved much additional research.

I owe a particular debt of gratitude to Professor L. J. MORDELL, who first introduced me to the Geometry of Numbers.

The proof-sheets have been read by Professors K. MAHLER, L. J. MORDELL and C. A. ROGERS. It is a pleasure to acknowledge their valuable help and advice both in detecting errors and obscurities and in suggesting improvements. Dr. V. ENNOLA has drawn my attention to several slips which survived into the second proofs.

I should also like to take the opportunity to thank Professor F. K. SCHMIDT and the Springer-Verlag for accepting this book for their celebrated yellow series and the Springer-Verlag for its readiness to meet my typographical whims.

Cambridge, June, 1959 J. W. S. CASSELS

Contents

Notation

An effort has been made to distinguish different types of mathematical object by the use of different alphabets. It is not necessary to describe the scheme in full since an acquaintance with it is not presupposed. However the following conventions are made throughout the book without explicit mention.

Bold Latin letters (large and small) always denote vectors. The dimensions is n, unless the contrary is explicitly stated: and the letter n is not used otherwise, except in one or two places where there can be no fear of ambiguity. The co-ordinates of a vector are denoted by the corresponding italic letter with a suffix $1, 2, \ldots, n$. If the bold letter denoting the vector already has a suffix, then that is put after the co-ordinate suffix. Thus:

$$\boldsymbol{a} = (a_1, \ldots, a_n)$$

$$\boldsymbol{b}_r = (b_{1r}, \ldots, b_{nr})$$

$$\boldsymbol{X}'_\varepsilon = (X'_{1\varepsilon}, \ldots, X'_{n\varepsilon}).$$

The origin is always denoted by $\boldsymbol{o}$. The length of $\boldsymbol{x}$ is

$$|\boldsymbol{x}| = (x_1^2 + \cdots + x_n^2)^{\frac{1}{2}}.$$

Sanserif Greek capitals, in particular $\mathsf{\Lambda}$, M, N, $\mathsf{\Gamma}$, denote lattices.

The notation $d(\mathsf{\Lambda})$, $\Delta(\mathscr{S})$, $V(\mathscr{S})$ for respectively the determinant of the lattice $\mathsf{\Lambda}$ and for the lattice-constant and volume of a set $\mathscr{S}$ will be standard, once the corresponding concepts have been introduced.

Chapters are divided into sections with titles. These sections are subdivided, for convenience, into subsections, which are indicated by a decimal notation. The numbering of displayed formulae starts afresh in each subsection. The prologue is just subdivided into sections without titles, and it was convenient to number the displayed formulae consecutively throughout.