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MuPAD Tutorial

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Preface

This book explains the basic use of the software package called MuPAD and gives an insight into the power of the system. MuPAD is a so-called computer algebra system, which is developed mainly by Sciface Software and the MuPAD Research Group of the University of Paderborn in Germany.

This introduction addresses mathematicians, engineers, computer scientists, natural scientists and, more generally, all those in need of mathematical computations for their education or their profession. Generally speaking, this book addresses anybody who wants to use the power of a modern computer algebra package.

There are two ways to use a computer algebra system. On the one hand, you may use the mathematical knowledge it incorporates by calling system functions interactively. For example, you can compute symbolic integrals or generate and invert matrices by calling appropriate functions. They comprise the system's mathematical intelligence and may implement sophisticated algorithms. Chapters 2 through 15 discuss this way of using MuPAD.

On the other hand, with the help of MuPAD's programming language, you can easily add functionality to the system by implementing your own algorithms as MuPAD procedures. This is useful for special purpose applications if no appropriate system functions exist. Chapters 16 through 18 are an introduction to programming in MuPAD.

You can read this book in the standard way "linearly" from the first to the last page. However, there are reasons to proceed otherwise. This may be the case, e.g., if you are interested in a particular problem, or if you already know something about MuPAD.

For MuPAD beginners, we recommend to start reading Chapter 2, which gives a first survey of MuPAD. The description of the online help system in Section 2.1 is probably the most important part of this chapter. The help system provides information about details of system functions, their syntax, their calling parameters, etc. It is available online during a MuPAD session.

In the beginning, requesting a help page is probably your most frequent query to the system. After you have grown familiar with the help system, you may start to experiment with MuPAD. Chapter 2 demonstrates some of the most important system functions “at work.” You will find further details about these functions in later parts of the book or in the help pages. For a deeper understanding of the data structures involved, you may consult the corresponding sections in Chapter 4.

Chapter 3 discusses the MuPAD libraries and their use. They contain many functions and algorithms for particular mathematical topics.

The basic data types and the most important system functions for their manipulation are introduced in Chapter 4. It is not necessary to study all of them in the same depth. Depending on your intended application, you may selectively read only the passages about the relevant topics.

Chapter 5 explains how MuPAD evaluates objects; we strongly recommend to read this chapter.

Chapters 6 through 11 discuss the use of some particularly important system functions: substitution, differentiation, symbolic integration, equation solving, random number generation, and graphic commands.

Several useful features such as the history mechanism, input and output routines, or the definition of user preferences are described in Chapters 12 through 14. Preferences can be used to configure the system’s interactive behavior after the user’s fancy to a certain extent.

Chapters 16 through 18 give an introduction to the basic concepts of the MuPAD programming language.

MuPAD provides algorithms which can handle a large class of mathematical objects and computational tasks related to them. Upon reading this introduction, it is possible that you encounter unknown mathematical notions such as rings or fields. This introduction is not intended to explain the mathematical background for such objects. Basic mathematical knowledge is helpful but not mandatory to understand the text. Sometimes you may ask what algorithm MuPAD uses to solve a particular problem. The internal mode of operation of the MuPAD procedures is not addressed here: we do not intend to give a general introduction to computer algebra and its algorithms. The interested reader may consult text books such as, e.g., [GCL 92] or [GG 99].

This book gives an *elementary* introduction to MuPAD. Somewhat more abstract mathematical objects such as, e.g., field extensions, are easy to describe and to handle in MuPAD. However, such advanced aspects of the system are not discussed here. The mathematical applications that are mentioned in the

text are intentionally kept on a rather elementary level. This is to keep this text plain for readers with little mathematical background and to make it applicable at school level.

We cannot explain the complete functionality of MuPAD in this introduction. Some parts of the system are mentioned only briefly. It is beyond the scope of this tutorial to go into the details of the full power of MuPAD's programming language. You find these in MuPAD's help system available online during a MuPAD session.

This tutorial refers to MuPAD versions 3.0 and later. Since the development of the system advances continuously, some of the details described may change in the future. Future versions will definitely provide additional functionality through new system functions and application packages. In this tutorial, we mainly present the basic tools and their use, which will probably remain essentially unchanged. We try to word all statements in the text in such a way that they stay basically valid for future MuPAD versions.

Paderborn, March 2004

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