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Excursions into Combinatorial Geometry

With 264 Figures

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To our wives

Lilia, Angelika and Luba

with love and thanks

Preface

Geometry undoubtedly plays a central role in modern mathematics. And it is not only a physiological fact that 80 % of the information obtained by a human is absorbed through the eyes. It is easier to grasp mathematical concepts and ideas visually than merely to read written symbols and formulae. Without a clear geometric perception of an analytical mathematical problem our intuitive understanding is restricted, while a geometric interpretation points us towards ways of investigation.

Minkowski's convexity theory (including support functions, mixed volumes, finite-dimensional normed spaces etc.) was considered by several mathematicians to be an excellent and elegant, but useless mathematical device. Nearly a century later, geometric convexity became one of the major tools of modern applied mathematics. Researchers in functional analysis, mathematical economics, optimization, game theory and many other branches of our field try to gain a clear geometric idea, before they start to work with formulae, integrals, inequalities and so on. For examples in this direction, we refer to [Mal] and [B-M 2].

Combinatorial geometry emerged this century. Its major lines of investigation, results and methods were developed in the last decades, based on seminal contributions by O. Helly, K. Borsuk, P. Erdös, H. Hadwiger, L. Fejes Tóth, V. Klee, B. Grünbaum and many other excellent mathematicians. Many of the open questions in this field can be explained in simple terms, even to non-mathematicians. But the solutions to these questions are often so complicated that every problem resolved is a major event.

It is impossible to give a complete list of references to this area, but the following books and surveys cover the major developments in combinatorial geometry: [D-G-K], [Grü 3,7,8], [H-D-K], [B-G 1,2], [B-SP 3], [Ec], [Rea], [Si], [SV 4], [B-So], chapters D and E in [C-F-G], [Be 3], [E], [Schm], [G-G-L], and [P-A]. We view the present book as a natural continuation of [B-G 1] and [B-SP 3]. The wider field of geometric convexity is covered for the most part in [Bl 2], [Eg 2,3], [Kl 2], [Ha 5], [Grü 6], [T-W], [G-W 1,2], [Schn], [B-M-S-W], [Zi], [Gar], [Tho] and several other publications.

Our book contains a collection of characteristic problems, tied together by several concepts: d-convexity, H-convexity, generalizations of the classical Helly theorem and its relatives, the Helly dimension of convex bodies, different covering and illumination problems etc. In addition, we introduce a new class of convex bodies, namely the family of belt bodies. The notion of a "belt body" is a natural generalization of the notion "zonoid". The belt bodies are dense in the set of all compact, convex bodies, whereas the zonoids are not even dense in the set of centrally symmetric, compact, convex bodies. It is shown that some difficult questions from combinatorial geometry can be completely resolved for belt bodies.

Our book can be used for different purposes. First, it is suitable for advanced undergraduate and graduate classes. With this audience in mind, we have amply illustrated the material with figures and provided numerous exercises. We hope readers will find this book stimulating to their geometric insight. Second, the book may serve as a reference to the current state of the art in some fields belonging to combinatorial geometry. Finally, specialists in these fields will find a collection of open problems in the last chapter.

A diagram at the beginning of the book illustrates the interdependence of the sections, and in addition some special "excursions" are described by section numbers. Note furthermore that some sections contain passages of survey character, that may be of less relevance, in first reading, to students. This applies in particular to sections 31 and 34.

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V. Boltyanski, H. Martini, and P.S. Soltan

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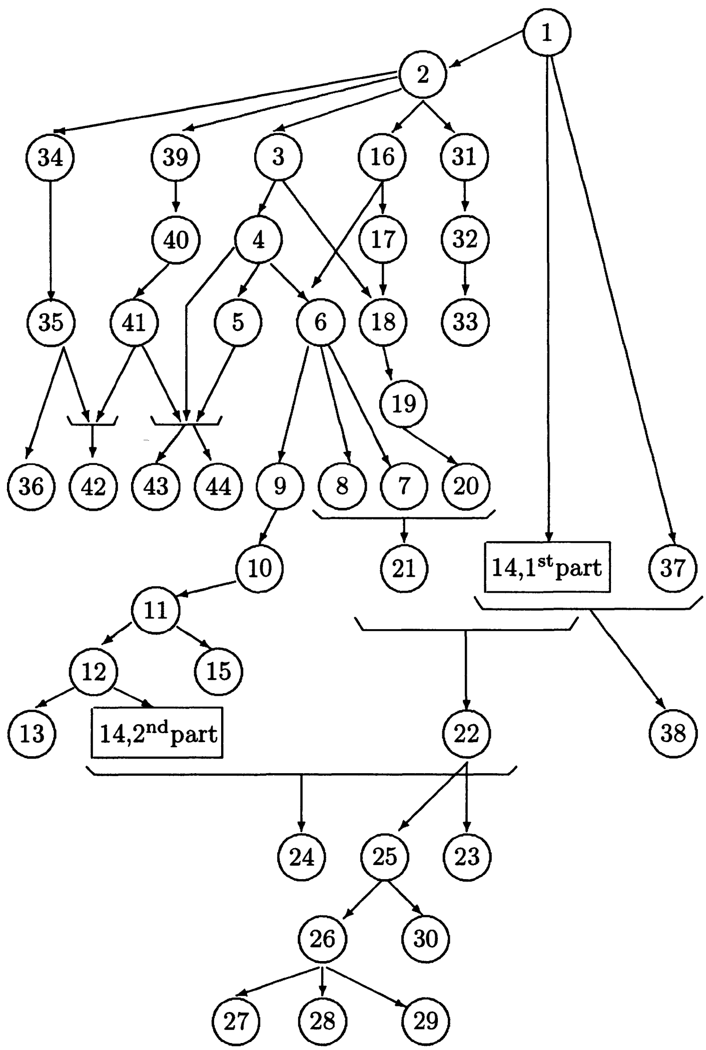
Dependence of sections



Some excursions

d-convexity:

Sections 1, 2, 3, 4, 6, 9, 10, 11, 12, 13, 14, 15
(and the section 24 after studying H-convexity).

H-convexity:

Sections 1, 2, 3, 4, 5, 6, 7, 8, 14 (1st part), 16, 17, 18, 19, 20, 21, 22, 23
(and the section 24 after studying d-convexity).

Szökefalvi-Nagy problem:

Sections 1, 2, 3, 4, 5, 6, 7, 8, 14 (1st part), 16, 17, 18, 19, 20, 21, 22, 25, 26, 27, 28, 29, 30 (and, in addition, sections 39, 40, 41, 43).

Borsuk problem:

Sections 1, 2, 31, 32, 33.

Gohberg-Markus-Hadwiger problem:

Sections 1, 2, 34, 35, 36
(and, in addition, sections 39, 40, 41, 42).

Inner illumination problem:

Sections 1, 2, 14 (1st part), 37, 38.

Minimal fixing systems:

Sections 1, 2, 3, 4, 16, 39, 40, 41, 44.