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Symbolic Dynamics

One-sided, Two-sided and Countable State Markov Shifts

With 65 Figures

Springer

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Preface

Nearly one hundred years ago Jacques Hadamard used infinite sequences of symbols to analyze the distribution of geodesics on certain surfaces. That was the beginning of symbolic dynamics. In the 1930's and 40's Arnold Hedlund and Marston Morse again used infinite sequences to investigate geodesics on surfaces of negative curvature. They coined the term symbolic dynamics and began to study sequence spaces with the shift transformation as dynamical systems. In the 1940's Claude Shannon used sequence spaces to describe information channels. Since that time symbolic dynamics has been used in ergodic theory, topological dynamics, hyperbolic dynamics, information theory and complex dynamics. Symbolic dynamical systems with a finite memory are studied in this book. They are the topological Markov shifts. Each can be defined by transition rules and the rules can be summarized by a transition matrix. The study naturally divides into two parts. The first part is about topological Markov shifts where the alphabet is finite. The second part is concerned with topological Markov shifts whose alphabet is countably infinite. The techniques used in the two cases are quite different. When the alphabet is finite most of the methods are combinatorial or algebraic. When the alphabet is infinite the methods are much more analytic.

This book grew from notes for a graduate course taught at Wesleyan University in the fall of 1994 and is intended as a graduate text and as a reference book for mathematicians working in related fields. Each chapter begins with an overview and then is divided into sections. At the end of each section are exercises. At the end of each chapter are historical notes and references.

The first section of the first chapter contains the beginning definitions of the spaces with a finite alphabet and the shift transformation. The second section is made up of examples. The examples show how topological Markov shifts arise in other areas of dynamics and in information theory. There are maps of the interval, Complex quadratic maps, the horseshoe, a toral automorphism and a channel code. The third section contains a short discussion of Perron-Frobenius theory for nonnegative, square matrices. The last section explains the basic dynamical properties of the shift transformation acting on a topological Markov shift. These properties include the topological entropy, the zeta function and a characterization of the continuous maps to be studied.

The second chapter is about homeomorphisms between topological Markov shifts which commute with the shift transformations. Such a map is called a topological conjugacy and when one exists between two topological Markov shifts they are said to be to be topologically conjugate. In the first section we see that a topological conjugacy can be decomposed into a sequence of elementary conjugacies. This leads to the development of an algorithm to determine when two one-sided topological Markov shifts are conjugate. We note that it is not known whether or not it is possible for such an algorithm to exist in the two-sided setting. In the second section shift-equivalence for two-sided topological Markov shifts is defined as an invariant for conjugacy. This is an algebraic relationship between transition matrices. This relationship is exploited to show relationships between the Jordan forms of the matrices and the dimension groups defined by the matrices.

Chapter three is about automorphisms of topological Markov shifts. An automorphism is a homeomorphism from a topological Markov shift to itself which commutes with the shift transformation. The automorphisms of a shift form a group under composition. The first section begins with a striking result which shows the automorphism group of the one-sided sequences on two symbols is isomorphic to $\mathbb{Z}/2\mathbb{Z}$ while the automorphism group of the two-sided sequences on two symbols contains every finite group. In the second section automorphisms are examined using the decomposition of conjugacies developed in chapter two. This allows us to show the automorphism groups of one-sided topological Markov shifts are generated by elements of finite order. It has recently been shown there are two-sided topological Markov shifts where the automorphism group is not generated by elements of finite order together with the shift transformation. The third section contains a discussion of subgroups of automorphism groups. This includes a discussion of finite subgroups and some observations about finitely presented groups. The fourth section contains observations about how the automorphisms act on the space and induce an automorphism of the dimension group. The decomposition of a conjugacy from chapter two is used to show how an automorphism of the space induces an automorphism of the dimension group.

The fourth chapter is about embeddings and factor maps. A factor map is a continuous map from one topological Markov shift onto another which commutes with the shift transformations. In the first section we see that factor maps fall into two distinct categories. They are either uniformly finite-to-one or uncountable-to-one on most points. Uniformly finite-to-one factor maps are examined in the second and third sections. A number of necessary conditions are developed for the existence of a uniformly finite-to-one factor map between two topological Markov shifts. These include conditions on the Jordan form of the transition matrices and conditions on some of the Bowen-Franks groups. In the fourth section necessary and sufficient conditions are developed for one topological Markov shift to be conjugate to a subsystem of another and for one topological Markov shift to be an unbounded-to-one factor of another.

Chapter five is about almost-topological conjugacy of topological Markov shifts. The first section is about reducible Markov shifts. The second section contains a description of the notion of almost-topological conjugacy and then develops a necessary and sufficient condition for two topological Markov shifts to be almost-topologically conjugate. The condition is that the topological entropy and the period of the two Markov shifts be the same.

Chapter six contains some additional topics. The first section contains a discussion of sofic systems. Sofic systems form a larger class of symbolic systems than the topological Markov shifts. A symbolic system on finitely many symbols is sofic if it can be obtained as the continuous image of a topological Markov shift. Markov measures on topological Markov shifts and the measure of maximal entropy are discussed in section two. Section three is about Markov subgroups. A Markov subgroup is a two-sided topological Markov shift with a group structure which makes the shift transformation an automorphism. There is a structure theorem for Markov subgroups and they are completely characterized up to topological conjugacy. A cellular automata is a continuous map from a topological Markov shift into itself. These are briefly discussed in section four. In the fifth section we examine a type of channel code used in data storage devices. These are called run-length limited codes. The constraints are described by finite transition rules and so the techniques previously developed are applicable. A method is developed to construct channel codes which meet certain encoding and decoding requirements.

The second part of this book is about topological Markov shifts with a countably infinite alphabet. There is far less known about these shifts than there is about finite state, topological Markov shifts. To examine countable state Markov shifts it is first necessary to extend the Perron-Frobenius theory for nonnegative, finite, square matrices to nonnegative, square, countably infinite matrices. This requires a considerable amount of work. First, the matrices are divided into three classes. Roughly speaking the classes correspond to how well the Perron- Frobenius Theorem can be approximated. This analytic division will also be mirrored in the dynamics of the Markov shifts the matrices define. Next the Perron-Frobenius Theorem is generalized and the differences in the classes of matrices is explored. The methods are very different than for finite matrices. Next there are a number of examples where the different types of behavior are illustrated.

In the second section countable state, topological Markov shifts are defined using countable transition matrices. The spaces for the Markov shifts are not compact and it leads to a number of difficulties. Topological entropy can be formulated in terms of the matrices, metrics, compactifications or invariant measures. Several of these formulations are given and compared. Finally, we examine some invariant measures and see that a form of the variational principle holds for a countable state topological Markov shift. We conclude by showing a countable state Markov shift has a maximal measure if and only if the transition matrix meets certain recurrence conditions.

Kinnakeet, North Carolina *Bruce Kitchens*
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