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Linear Programming Duality

An Introduction to Oriented Matroids

With 40 Figures

Springer-Verlag Berlin Heidelberg GmbH

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Mathematics Subject Classification (1991): 05 B 35, 90 C 05, 90 C 27, 90 C 48

ISBN 978-3-540-55417-2 ISBN 978-3-642-58152-6 (eBook)
DOI 10.1007/978-3-642-58152-6

Library of Congress Cataloging-in-Publication Data
Bachem, A. Linear programming duality: an introduction to oriented matroids / Achim Bachem, Walter Kern. p. cm. - (Universitext) Includes bibliographical references and index. ISBN 978-3-540-55417-2 1. Matroids. 2. Linear programming. I. Kern, Walter, 1957- . II. Title. QA166.6.B33 1992 511'.6-dc20 92-14021

Originally published by Springer-Verlag Berlin Heidelberg New York in 1992

Typesetting: Camera ready by author
41/3140-5 4 3 2 1 0 – Printed on acid-free paper

Preface

The main theorem of Linear Programming Duality, relating a "primal" Linear Programming problem to its "dual" and vice versa, can be seen as a statement about sign patterns of vectors in complementary subspaces of $\mathcal{R}^n$.

This observation, first made by R.T. Rockafellar in the late sixties, led to the introduction of certain systems of sign vectors, called "oriented matroids". Indeed, when oriented matroids came into being in the early seventies, one of the main issues was to study the fundamental principles underlying Linear Programming Duality in this abstract setting.

In the present book we tried to follow this approach, i.e., rather than starting out from ordinary (unoriented) matroid theory, we preferred to develop oriented matroids directly as appropriate abstractions of linear subspaces. Thus, the way we introduce oriented matroids makes clear that these structures are the most general - and hence, the most simple - ones in which Linear Programming Duality results can be stated and proved. We hope that this helps to get a better understanding of LP-Duality for those who have learned about it before und a good introduction for those who have not.

The present book arose from an early draft on polyhedral theory and two graduate courses held at Köln university in 1989/1990. No specific prerequisits are assumed. Basically, all we require is some familarity with linear algebra. The book is intended as an introduction to oriented matroid theory for the non-expert. Therefore we restricted ourselves to a thorough discussion of rather elementary results such as linear duality theorems and abstract polyhedral theory.

The only more advanced topic we treat is the topological realization of oriented matroids due to J.Folkman and J.Lawrence, resp. J.Edmonds and A.Mandel. A further reason for restricting ourselves in this way was the fact that at about the same time as this book will appear, a much more comprehensive treatment of oriented matroids by A.Björner, M.Las Vergnas, B.Sturmfels, N.White and G.Ziegler including an extensive discussion of more advanced topics will be available. Instead of repeating a reference to this work at the end of each chapter of our book we would like to recommend it here, once and for all, to the interested reader.

We are grateful to several colleagues for many stimulating discussions on the topic of this book. We are particularly grateful to Michael Hofmeister und Winfried Hochstättler who read the "beta version" of the text and provided extensive comments and suggestions. We have worked on this book and written the text at various universities. We acknowledge, in particular, the support of the German Research Association (DFG), the universities of Bonn, Köln and Twente.

Our special thanks are due to Frank Neuhaus for the efficient and careful typing the text in TEX.

Köln, October 1991

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Notation

We use standard notation throughout as far as possible and there are only very few things that are worth being mentioned explicitely:

Quantifiers are denoted by $\exists$ and $\forall$. "$\subseteq$" stands for (set theoretical) containment and "$\subset$" always means proper containment. Disjoint union will be denoted by "$\dot\cup$".

If A is an $m \times n$ matrix, then
$A_{i\cdot}$, $i \in \{1, \ldots, m\}$ denotes the i-th row of A and
$A_{\cdot j}$, $j \in \{1, \ldots, n\}$ denotes the j-th column of A.

More generally, if $I \subseteq \{1, \ldots, m\}$ and $J \subseteq \{1, \ldots, n\}$, then $A_{I\cdot}$ denotes the matrix made up by the rows indexed by I and $A_{\cdot J}$ denotes the matrix made up by the columns indexed by J. A_{IJ} denotes the submatrix of elements indexed by $(i, j) \in I \times J$. The transpose of a matrix A and a vector x are denoted by A^T and x^T, resp. Sometimes, transposition of vectors is omitted, in case no misunderstanding is possible. Thus, usually, vectors are implicitely assumed to be column vectors, but they may be used as row vectors sometimes without making the transposition explicitely. If A and B are subsets of a vectorspace, then $A + B$ denotes the set $\{a + b \mid a \in A,\ b \in B\}$. If Λ is a set of scalars, then $\Lambda A = \{\lambda a \mid \lambda \in \Lambda,\ a \in A\}$.

The set of all subsets of a set E is denoted by 2^E.
$\mathbb{Z}$ denotes the set of integers and $\mathbb{N}$ denotes the the set of positive integers.