Part VII

Nonreflecting Artificial Boundary Conditions for Replacing the Rejected Equations with Lacunas

In many problems of mathematical physics the desired function or vector function $u(x,t)$ is defined in the entire three-dimensional space $\mathbb{R}^3$ of points $x = (x_1, x_2, x_3)$ except for, perhaps, some bounded domain Q. The domain of definition of $u(x,t)$ is denoted by $D = \mathbb{R}^3 \setminus Q$.

To be definite, we assume that Q lies entirely inside the sphere $x_1^2 + x_2^2 + x_3^2 \leq q^2$ of radius $q < 1$. Now let the solution satisfy the condition $u(x,t) = 0$ for $t \leq 0$ and some boundary conditions on ∂Q. We also assume that the differential equations under study are linear and homogeneous outside the unit sphere. Suppose that we are interested in the solution $u(x,t)$ only inside the unit sphere. In this case, we wish to construct some boundary conditions on the unit sphere (which separates the computational domain where we want to know the solution from the rest of the domain of definition D of the solution), so that these boundary conditions would equivalently replace the equations rejected outside the computational domain, i.e., outside the unit sphere S_1. In what follows, we shall construct nonreflecting artificial boundary conditions (NRABCs) for the case in which the differential equation of the problem is the wave equation outside the unit sphere. This will be done by the method of difference potentials. In the proposed construction of NRABCs, an auxiliary difference problem must be solved. Further, we present an economical algorithm for solving this problem. The algorithm is based on the fact that solutions of wave equations have lacunas. We shall discuss this property in Chap. 1.

The problem will be studied in the difference setting. We shall construct nonreflecting artificial boundary conditions so that the computer time per one time step does not increase with the number of the step. The order of the number of arithmetical operations corresponding to the number of computational points cannot be decreased, since this order coincides with the order of the number of computational points.

Note that in the construction of nonreflecting artificial boundary conditions and of an algorithm for computing these conditions we essentially use not the special forms of the wave equation and of the computational subdomain, but only the fact that the solution of the equation (or that of the system of equations which we reject by replacing them with the help of NRABCs on the boundary of the computational subdomain) can have lacunas. Hence we can hope that, without principal difficulties, our arguments can be generalized to the cases in which the solution of the original problem outside the computational domain satisfies the Maxwell or Lamé systems, whose solutions also have lacunas, rather than the wave equation.

In Chap. 1 we study the lacunary property of solutions to the wave equation and reduce the problem of constructing NRABCs to solving an auxiliary difference Cauchy problem. In Chap. 2 we construct an economical algorithm for solving this auxiliary Cauchy problem and consider the results of numerical experiments. These results confirm the effectiveness of our approach in the cases under study.