Part III

A General Scheme of the Method of Difference Potentials for the Numerical Solution of Differential and Difference Boundary-Value Problems of Mathematical Physics

We shall consider differential equations or systems of equations of the form

$$L_{D\bar{D}}u_{\bar{D}} = f_D, \tag{I}$$

subject to additional conditions of the form

$$l_{Q\bar{D}}u_{\bar{D}} = \varphi_Q. \tag{II}$$

When $Q = \partial D = \Gamma$ and the result of the application of the operator $l_{Q\bar{D}}$ depends on the values of $u_{\bar{D}}$ only in an arbitrarily small ε-neighborhood of the boundary Γ, problem (I), (II) is a boundary-value problem.

In Chap. 1 we shall describe a general scheme for the numerical solution of problem (I), (II) by reducing this problem to boundary equations with projections and by further discretizing these boundary equations with the aid of difference potentials. In Chap. 2 we illustrate the main constructions by means of certain internal and external problems and we construct a difference potential for the numerical solution of problems involving the Laplace equation in a domain with a cut. In Chap. 3 we introduce a general scheme for the so-called spectral approach to the approximation of the boundary conditions (II). This approach is based on the use of difference potentials.

The spectral approach allows us to overcome typical difficulties associated with the difference approximation of conditions (II) in the case of complicated boundary conditions and also in the case of grids not conforming to the boundary of the domain D. In Chap. 3 we also consider a general scheme for calculating the solution of the difference boundary-value problem which is based on the approximation of problem (I), (II) by the method of difference potentials.