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Mathematical Concepts of Quantum Mechanics

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Preface

The first fifteen chapters of these lectures (omitting four to six chapters each year) cover a one term course taken by a mixed group of senior undergraduate and junior graduate students specializing either in mathematics or physics. Typically, the mathematics students have some background in advanced analysis, while the physics students have had introductory quantum mechanics. To satisfy such a disparate audience, we decided to select material which is interesting from the viewpoint of modern theoretical physics, and which illustrates an interplay of ideas from various fields of mathematics such as operator theory, probability, differential equations, and differential geometry. Given our time constraint, we have often pursued mathematical content at the expense of rigor. However, wherever we have sacrificed the latter, we have tried to explain whether the result is an established fact, or, mathematically speaking, a conjecture, and in the former case, how a given argument can be made rigorous. The present book retains these features.

Prerequisites for this book are introductory real analysis (notions of vector space, scalar product, norm, convergence, Fourier transform) and complex analysis, the theory of Lebesgue integration, and elementary differential equations. These topics are typically covered by the third year in mathematics departments. The first and third topics are also familiar to physics undergraduates. Those unfamiliar with Lebesgue integration can think about Lebesgue integrals as if they were Riemann integrals. This said, the pace of the book is not a leisurely one and requires, at least for beginners, some amount of work.

Even in dealing with mathematics students we have found it useful, if not necessary, to review basic mathematical notions such as the spectrum of an operator, and the Gâteaux or variational derivative, which we needed for the course. Moreover, to make the book relatively self-contained, we recall and sometimes discuss the basic notions mentioned above. As a result, the text is interspersed with mathematical supplements which occupy in total about a third of the material. A mathematically sophisticated reader can skim through them, or skip them altogether, and concentrate on physical applications. On the other hand, readers familiar with the physical content of quantum mechanics, and who would like to enhance their mathematics, could concentrate on those detours and consider the physics chapters as an application of the mathematics in a familiar setting.

Though we tried to increase the complexity of the material gradually, we were not always successful, and first in Chapter 8, and then in Chapter 13, there is a leap in the level of sophistication required from the reader.

This book consists of fifteen main chapters and one supplementary chapter, Chapter 16. The latter chapter is more technical than the preceding material. We did not include many standard topics which are well-covered elsewhere. These topics are referenced in Chapter 17, where we also give some comments on the literature and further reading.

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