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Atoms, Molecules and Optical Physics 2

Molecules and Photons – Spectroscopy and Collisions

Springer

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To my Wife Erika
IVH

To my Wife Gudrun
CPS

Preface

The two textbooks on Atomic, Molecular and Optical (AMO) physics presented here aim at providing something like the canonical knowledge of modern atomic and molecular physics together with a first entry into optical physics and quantum optics. All of these topics constitute a vital area of active and highly productive research in physics. And in spite of, or perhaps even because of its remarkable history the field continues to constitute an indispensable basis for any more profound understanding of nearly all branches of modern physics, physical chemistry and partially even biological and material sciences. Specifically the latter appear to become more and more based on genuine molecular concepts.

We want to address on the one hand advanced students of physics and physical chemistry, who have to study these topics within their respective curricula. At the same time, however, these textbooks should be useful to all those who discover in different contexts that they miss some essential basics from this field and who seek for suitable means to acquire that knowledge. Of course, we also address quite specifically Ph.D.-students or young researchers who start for the first time their own activities in the field – or just want to know more about it. They will find here reliable knowledge and stimulating challenges for their own work. We thus have tried not only to provide the essential basics for working with these topics, but whenever possible also to inform the interested reader about the present state-of-the-art and to allow her or him a glimpse on today's cutting edge research.

The general remarks as well as details about formats, notation, units, and typography outlined in the preface to Vol. 1 (HERTEL and SCHULZ 2014) are equally valid for the present Vol. 2. In the following we just give a guide through the contents, as the readers will find many topics and details far beyond the standard textbooks and routine teachings on AMO science.

Chapter 1 resumes the discussion of light (comprising in the broadest sense the whole electromagnetic spectrum) and photons, which are key themes of these two volumes. The focus is here on *lasers* (one of the most important tools of modern AMO physics), Gaussian beams, polarization and nonlinear processes. In the following Chap. 2 the emphasis is on the properties of *photons* and *coherence*. In addition to discussing some basics of quantum optics and its applications, we also

complete now the theory of photon induced transitions (Chap. 4, Vol. 1) and introduce *field quantization* which allows us to treat spontaneous emission.

After these preparations we are ready to enter into *molecular physics* and modern *molecular spectroscopy*. We start in Chap. 3 with diatomic molecules, the most simple prototypes, and enhance our view with 'real' polyatomic molecules in Chap. 4, including a brief excursion into the subject of symmetries. While along this way we have already encountered various comparatively simple examples of molecular spectroscopy, Chap. 5 leads us deep into a variety of sophisticated *modern methods of spectroscopy*. The field is dominated today by laser based methods, but we also gain some insights e.g. into the possibilities of photoelectron spectroscopy.

Three quite detailed Chaps. 6–8 are devoted to the present status in the physics of electronic, atomic, molecular and ionic collisions (including collisional ionization) – a topic of great practical importance and with demanding intellectual challenges, both from an experimental and theoretical view point. Chapter 9 gives a down to earth manual for using the *density matrix*. It also gives a brief look on the theory of measurement, including a powerful method to analyze radiation patterns from anisotropically populated mixtures of excited states. Finally, making use of these concepts, an introduction of the *optical* BLOCH *equations* is given in Chap. 10, which again addresses many exciting facets of quantum optics with interesting examples.

A possible extension of the two volumes published now is under consideration. Such a Vol. 3 would approach modern research even more closely and illuminate some particularly *hot* and rapidly developing areas such as ultra-cold matter and quantum gases, ultrafast dynamics, attosecond physics, cluster spectroscopy and similar themes.

We wish all our readers an exciting and stimulating reading as well as efficient understanding and successful learning. In several *readings*, we have tried to produce text, mathematical formulas and figures as free from errors as possible. Clearly, this can only be an approximation process. Thus, we encourage our readers to kindly communicate any critical comments, errors or simply even typos which they may discover – and to make suggestions for improvements wherever it appears advisable. We shall correct such errors at the web-site http://staff.mbi-berlin.de/AMO/book-homepage/ if and as soon as they become known to us.

As further reading we recommend for comparison or for a deeper look into some specialties the textbooks listed below.

Berlin Adlershof
January 2014

Ingolf V. Hertel
Claus-Peter Schulz

Acronyms and Terminology

AMO: 'Atomic, molecular and optical', physics.

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Of course, there all other sources are documented which we have used for informations and which have provided data which we have used to generate the figures in these books.

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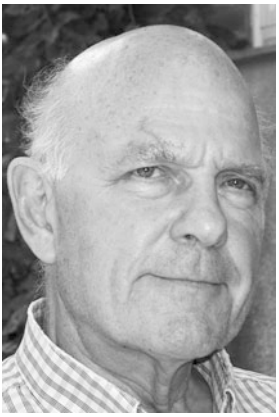

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