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*Series Editors*

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# Geometry of the Standard Model of Elementary Particles



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# Preface

The standard model is the currently accepted theory of particles and interactions, describing them in a manner that is both coherent and consistent with experimental data. This book deals only with the *classical part* of the standard model, obtained by ignoring its aspects related to field quantization. Despite its shortcomings as a description of nature, such a classical approach offers a simple yet far-reaching insight into the structure of particle theories (see end of §6.7).

The reader need not be familiar with theoretical physics, but is expected to know the basic facts about Riemannian manifolds and connections in vector bundles, as presented, e.g., in the appropriate introductory chapters of [Besse 1987], [Kobayashi and Nomizu 1963], or [Milnor and Stasheff 1974]. The approach used is based on geometric language (coordinate-free, whenever possible), and should be easily accessible to mathematicians, as well as physics students with some background in geometry.

The chapters and sections marked by asterisks are considered optional and may be skipped, leaving an essentially self-contained, 67-page-long core of the text. It covers the most important topics (briefly summarized in [Derdzinski 1991a]), such as the Yang-Mills description of interactions, the quark and electroweak models, and the SU(5) grand unification. The optional chapters deal with further material, including the geometry of (iso)spin, particle invariants, unitary symmetry, the Cabibbo angle, Lagrangian densities, and the Higgs mechanism.

The purpose of this book is to fill what I perceive as a gap in the available literature. There exist many excellent accounts of these and related topics, both in the form of physics books and mathematical texts (see the reference list). However, while the former usually require much training in physics, and describe geometric constructions in terms of coordinates, the latter's emphasis is on mathematics and generality. My objective has been to present the classical version of the theory believed to reflect physical reality, by means of a consistently geometric exposition that avoids extensive generalizations, and assumes no prerequisite knowledge of physics.

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*Andrzej Derdzinski*

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