

# Lecture Notes in Operations Research and Mathematical Economics

Edited by M. Beckmann, Providence and H. P. Künzi, Zürich

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## Mathematical Systems Theory and Economics I

Proceeding of an International Summer School  
held in Varenna, Italy, June 1 - 12, 1967

Edited by

H. W. Kuhn

Princeton University, Princeton, N. J./ USA

G. P. Szegö

University of Milano, Milano/Italy

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Springer-Verlag Berlin Heidelberg GmbH

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ISBN 978-3-540-04635-6      ISBN 978-3-642-46196-5 (eBook)  
DOI 10.1007/978-3-642-46196-5

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Library of Congress Catalog Card Number 70-81409 Title No. 3761

## Preface

The International Summer School on Mathematical Systems Theory and Economics was held at the Villa Monastero in Varenna, Italy, from June 1 through June 12, 1967. The objective of this Summer School was to review the state of the art and the prospects for the application of the mathematical theory of systems to the study and the solution of economic problems. Particular emphasis was given to the use of the mathematical theory of control for the solution of problems in economics. It was felt that the publication of a volume collecting most of the lectures given at the school would show the current status of the application of these methods.

The papers are organized into four sections arranged into two volumes: basic theories and optimal control of economic systems which appear in the first volume, and special mathematical problems and special applications which are contained in the second volume. Within each section the papers follow in alphabetical order by author.

The seven papers on basic theories are a rather complete representative sample of the fundamentals of general systems theory, of the theory of dynamical systems and the theory of control. The five papers on the application of optimal control to economic systems present a broad spectrum of applications.

The series of lectures given at the school by Prof. A. Strauss on the mathematical theory of control were published as a separate volume in this series, with title "An Introduction to Optimal Control Theory". Five interesting lectures given at the School by Prof. H. Scarf, E. Sheshinsky, R. Radner, J. Florentin and S. Mitter were not made available for publication. All other papers presented appear in this volume.

This International Summer School was financially supported by NATO. Fellowships to the participants were in addition given by various organizations, to whom we express our gratitude.

Princeton, N.J., USA

Milano, Italy

December 1968

H.W. Kuhn

G.P. Szegö

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### List of Contributors

- Albrecht, F., University of Illinois, Department of Mathematics, Urbana, Ill. / USA
- Beckmann, M. J., Institut für Ökonometrie und Unternehmensforschung der Universität,  
5300 Bonn / Germany
- Bhatia, N. P., Case Western Reserve University, Department of Mathematics,  
Cleveland, Ohio / USA
- Blikle, A. J., Polish Academy of Sciences, Mathematical Institute, Warszawa / Poland
- Castaing, C., Université de Montpellier, Division Mathématiques, Montpellier,  
Hérault / France
- Debreu, G., University of California, Department of Economics, Berkeley, Calif. / USA
- Dobell, A. R., Harvard University, Department of Economics, Cambridge, Mass. / USA
- Halanay, A., Bucharest University, Bucharest / Roumania
- Halkin, H., University of California, Department of Mathematics, San Diego, Calif. / USA
- Hildenbrand, W., University of California, Department of Economics, Berkeley, Calif. / USA  
and Studiengruppe für Systemforschung, 6900 Heidelberg / Germany
- Hillinger, C., Case Western Reserve University, Division of Organizational Sciences,  
Cleveland, Ohio / USA
- Ho, Y. C., Harvard University, Computation Laboratory, Cambridge, Mass. / USA
- Kalman, R. E., Stanford University, Department of Mathematics, Stanford, Calif. / USA
- Kuhn, H. W., Princeton University, Department of Mathematics, Princeton, New Jersey / USA
- Kulikowski, R., Polish Academy of Sciences, Institute of Automatic Control, Warszawa  
Poland
- Kurz, M., Stanford University, Institute for Mathematical Studies in the Social  
Sciences, Stanford, Calif. / USA
- Lee, E. B., Center for Control Sciences, Institute of Technology, University of  
Minnesota, Minneapolis, Minn. / USA
- Markus, L., University of Minnesota, Department of Mathematics, Minneapolis, Minn. / USA
- McFadden, D., University of California, Institute for International Studies,  
Berkeley, Calif. / USA
- Mesarovic, M. D., Case Western Reserve University, Systems Research Center,  
Cleveland, Ohio / USA
- Nagy, J., Technical University (CVUT), Department of Mathematics FEL, Technicka 2/1902,  
Prague 6-Deyjvice / Czechoslovakia
- Neustadt, L. W., University of Southern California, Department of Electrical Engineering,  
Los Angeles, Calif. / USA
- Rockafellar, R. T., University of Washington, Department of Mathematics, Washington,  
Seattle / USA
- Ryder, H. E., Jr., Brown University, Department of Economics, Providence, Rhode Island / USA

Shell, K., University of Pennsylvania, Department of Economics, Philadelphia, Pa. / USA

Straszak, A., Polish Academy of Sciences, Institute for Automatic Control,  
Warszawa / Poland

Yorke, J. A., University of Maryland, Department of Mathematics, College Park,  
Maryland / USA