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Analysis of Neural Networks

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To DORIS

PREFACE

The purpose of this work is a unified and general treatment of activity in neural networks from a mathematical point of view. Possible applications of the theory presented are indicated throughout the text. However, they are not explored in detail for two reasons : first, the universal character of neural activity in nearly all animals requires some type of a general approach; secondly, the mathematical perspicuity would suffer if too many experimental details and empirical peculiarities were interspersed among the mathematical investigation. A guide to many applications is supplied by the references concerning a variety of specific issues.

Of course the theory does not aim at covering all individual problems. Moreover there are other approaches to neural network theory (see e.g. Poggio-Torre, 1978) based on the different levels at which the nervous system may be viewed.

The theory is a deterministic one reflecting the average behavior of neurons or neuron pools. In this respect the essay is written in the spirit of the work of Cowan, Feldman, and Wilson (see sect. 2.2).

The networks are described by systems of nonlinear integral equations. Therefore the paper can also be read as a course in nonlinear system theory. The interpretation of the elements as neurons is not a necessary one. However, for vividness the mathematical results are often expressed in neurophysiological terms, such as excitation, inhibition, membrane potentials, and impulse frequencies.

The nonlinearities are essential constituents of the theory. Important phenomena such as hysteresis, limit cycles, pulses only occur with nonlinear systems, and obviously neurons are nonlinear elements.

The term "analysis" in the title is meant in the mathematical sense. In particular the statements marked as theorems are provable. Of course many interesting and important properties of nonlinear systems can only be investigated by numerical methods. On the other hand there is now rapid progress in nonlinear analysis, and neural network theory should profit from this development.

Proofs of theorems are only given when the author could not find a result in the literature including the theorem.
The book is organized as follows. In the first chapter the network equations are derived on the basis of the universal properties and interaction principles of neurons and their organization in neural tissues. The second chapter shows how our model is related to other models in the literature, in particular to some experimentally investigated networks. The analysis is started in chap.3 establishing mathematical conditions in order that the equations unambiguously determine the states of the (model) networks. All further chapters investigate the (temporal or stationary) behavior of the model-networks following from the equations. Chapters 4-6 are restricted to nets composed of finitely many elements (these may be single neurons or populations of neurons with spatially homogeneous behavior). The last three chapters exhibit modes of behavior necessarily dependent on the spatial extension of neural tissues, such as wave form activity or spatial patterns. The last section (9.4) establishes some connexions to other literature and applications.
Essential parts of chapters 1-3 appeared in my paper "Structures of excitation and inhibition" published in Vol.21 of this series. Section 6.3 contains a slight generalization of a result which appeared in the Journal of Mathematical Analysis and Applications. Chapters 1-6 essentially coincide with my "Habilitationsschrift" submitted to the Faculty of Biology at the University of Tübingen in January 1979.

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