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# Statistical Decision Theory

 Springer

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*To Mr. A. Horger and his colleagues*

# Preface

Making decisions is at the heart of all human activity, mundane as well as adventurous, for both livelihood and fun. It is an abstract process in which we try to anticipate what may happen if we choose one course of action or another, and weigh carefully which one is safer, more enjoyable and profitable, or will bear greater benefits of some other kind. The definition of statistics assumed in this volume is that of making decisions in the presence of uncertainty and with limited resources. In this perspective, any meaningful analysis should closely look at the consequences of the possible outcomes and of the recommendations based on them.

Uncertainty means that we may get it wrong. For example, even a competently executed test of a hypothesis may conclude with evidence against the null when in fact the null is valid. We do not allow a 5 % chance when crossing a road known to be busy—it makes sense to look at and listen to the traffic, appreciating the consequences of getting this everyday manouevre wrong. In a statistical analysis, the consequences of the errors we commit may or may not be as lop-sided as in this example, but their balance is a relevant factor in the analysis.

To my regret, I have come to this view only recently, prompted by my experiences in consulting in which some clients have dismissed the standard format of an analysis that concludes with a hypothesis test, a confidence interval, or an estimate, because such a statement requires further nontrivial processing (interpretation) to decide what to do: how to alter business practices, production settings, strategic goals, medical treatment regimes, and the like. I got the drift and re-discovered the work of Morris DeGroot, Tom Ferguson, Dennis Lindley and Jim Berger, and this volume is a document of my conversion—not to a Bayesian, but to regarding a decision as the ultimate goal of an analysis.

The volume has an unashamed partisan character; I would like to convert the reader to the perspective I present, or at least to appreciate its merits, together with its practical feasibility and the potential appeal to a client.

**Chapter 1** introduces the main ideas and terminology, discusses the established formats in which the results of statistical analyses are most frequently presented, and highlights their unsatisfactory features. **Chapters 2** and **3** deal with two elementary problems, estimation of the mean and variance of a normal random sample, when the consequences of errors with one sign differ from those with the other sign. The problem of choosing one of the options (courses of action) is also

addressed. [Chapter 4](#) introduces the Bayesian paradigm as a vehicle for incorporating prior information about the parameters involved in a model. These priors are informative and are regarded as important as the data. [Chapter 5](#) steps beyond the confines of the normal distribution and shows that decisions with samples from and statistics with some other distributions involve calculus not substantially more complex than in the normal case. [Chapters 6](#) and [7](#) deal with two applications, classification and small-area estimation, in which, I believe, the absence of a transparent discussion of losses (utilities) has led the development astray, sometimes in the direction of irrelevance. [Chapter 8](#) deals with study design. Its length and location in the volume do no justice to the importance of design in practice, but without having settled on the method of analysis its discussion would not be constructive.

All the evaluations described in this volume were made in R (R Development Core Team, 2009), with user-defined functions. Apart from working out all the examples, this also had the purpose of quality control of all the math expressions in the text. The entire library is available from [www.sntl.co.uk/Decide.html](http://www.sntl.co.uk/Decide.html). However, a reader may draw greater benefit from the volume by implementing some or all the algorithms himself or herself, exploring alternative graphical displays and assessing firsthand the programming effort required.

The original intention of the volume was to communicate the relevance, feasibility, and value of decision theory for the ‘everyday’ statistical problems to (statistical) professionals, but I paid attention during the revisions also to the potential of using the text in a graduate course in statistics. A good background in calculus and linear algebra is important and proficiency in R is a distinct advantage. With it, the text can be used in a graduate course in one academic quarter. The necessary background from other textbooks can be combined with this text in a semester. If a topic has to be dropped from the course, [Chaps. 6](#) or [7](#) are better choices than [Chap. 8](#).

I would like to thank Alex Ruiz de Villa Robert for insightful comments on parts of the manuscript, to Omiros Papaspiliopoulos for highly informative discussions, and to Springer-Verlag for the original idea, assistance with preparation of the manuscript, arranging helpful reviews of a draft manuscript and smooth and timely production of the monograph. The project was partly supported by a grant from the Spanish Ministry of Science and Innovation.

Barcelona, April 2013

# Contents

<b>1</b>	<b>Introduction</b>	1
1.1	The Role of Statistics	2
1.2	Preliminaries	4
1.3	Estimation	6
1.4	Assessing an Estimator	8
1.5	Confidence Intervals	9
1.6	Hypothesis Testing	10
1.7	Loss Function	12
1.8	Problems, Exercises and Suggested Reading	14
	References	15
<b>2</b>	<b>Estimating the Mean</b>	17
2.1	Estimation with an Asymmetric Loss	17
2.2	Numerical Optimisation	19
2.3	Plausible Loss Functions	21
2.4	Other Classes of Loss Functions	23
2.4.1	LINEX Loss	24
2.5	Comparing Two Means	26
2.6	Problems, Exercises and Suggested Reading	29
	References	31
<b>3</b>	<b>Estimating the Variance</b>	33
3.1	Unbiased and Efficient Estimation	33
3.2	Loss Functions for Variance Estimators	35
3.3	Variance Versus a Constant	39
3.3.1	Decision with Utilities	40
3.3.2	Multiplicative Loss	41
3.4	Estimating the Variance Ratio	43
3.5	Problems, Exercises and Suggested Reading	46
	References	47

<b>4</b>	<b>The Bayesian Paradigm</b> . . . . .	49
4.1	The Bayes Theorem. . . . .	49
4.2	Comparing Two Normal Random Samples . . . . .	53
4.3	Decision with Estimated $\sigma^2$ . . . . .	58
4.4	Problems, Exercises and Suggested Reading . . . . .	63
	References . . . . .	64
<b>5</b>	<b>Data from Other Distributions</b> . . . . .	65
5.1	Binary Outcomes. . . . .	65
5.2	Poisson Counts . . . . .	70
5.3	Continuous Distributions: Transformations and Mixtures . . . . .	74
5.4	Problems, Exercises and Suggested Reading . . . . .	76
	References . . . . .	77
<b>6</b>	<b>Classification</b> . . . . .	79
6.1	Introduction . . . . .	79
6.2	Normally Distributed Marker . . . . .	80
6.3	Markers with Other Distributions . . . . .	84
6.3.1	Markers with $t$ Distribution . . . . .	84
6.3.2	Beta Distributed Markers . . . . .	86
6.3.3	Gamma Distributed Markers. . . . .	87
6.4	Looking for Contaminants . . . . .	88
6.5	Problems, Exercises and Suggested Reading . . . . .	91
	References . . . . .	93
<b>7</b>	<b>Small-Area Estimation</b> . . . . .	95
7.1	Composition and Empirical Bayes Models . . . . .	96
7.2	Estimation for a Policy. . . . .	98
7.3	Application . . . . .	102
7.3.1	Limited Budget. . . . .	105
7.4	Conclusion . . . . .	106
7.5	Appendix: Estimating $\sigma_B^2$ . . . . .	107
7.6	Problems, Exercises and Suggested Reading . . . . .	108
	References . . . . .	109
<b>8</b>	<b>Study Design</b> . . . . .	111
8.1	Sample Size Calculation for Hypothesis Testing . . . . .	112
8.2	Sample Size for Decision Making . . . . .	112
8.3	Problems, Exercises and Suggested Reading . . . . .	118
	References . . . . .	119
	<b>Index</b> . . . . .	121