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Vladimir M. Fomin

Editor

Physics of Quantum Rings

 Springer

Editor

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Foreword

Physics of Solids developed into an independent discipline at the end of the 30ies of last century with the formulation of the theory of electronic band structure. Boundary condition for the validity of this theory is the assumption of an infinitely extended crystal showing no defects, interfaces or surfaces. Almost at the same time the quantum mechanical problem of a particle in a one-dimensional potential well was solved for the first time.

Semiconductors were recognized as an important class of solids only a decade later, although the first roots go back to the 19th century when Ferdinand Braun, well known as the inventor of the cathode ray tube, wrote in 1874 his thesis on “Current Conduction through Sulfur-Metals”. The subject of this thesis got much later the name “Schottky Diode”. The developments of the transistor by William Shockley and his coworkers starting 1947 and of III–V-compounds by Heinrich Welker already in 1951 present landmarks decisive for the advancement of modern multi-targeted technologies enabling today solar cells, microprocessors or semiconductor lasers, to mention a few device groups having diffused into our daily life. Indeed it is unthinkable to live without such devices enabling, in particular, modern communication technologies.

Heterostructures, layered semiconductor/semiconductor or semiconductor/insulator structures like Si/SiO₂, were essential parts of devices like transistors from the very beginning. With the advent of III–V-based heterostructures, presenting the basis for light emitting, but also highly efficient light harvesting devices, the materials basis for a wealth of devices and systems broadened enormously and the scientific community embarked to explore “chemical engineering” in a very systematic way. Twice Nobel prizes were awarded for the physics of Si- and III–V-based heterostructures in 1985 and 2000. The limits of combining materials of varying chemical composition on top of each other were discovered to be controlled by the variation of lattice constants between different materials. If this difference is too large, defects like dislocations develop and the device properties degrade. Thus, the original enthusiasm on “chemical engineering” was fast decaying at the end of the 80ies of last century and almost entirely “lattice-matched heterostructures” were

thought to be useful, restricting enormously the range of structures being available for III–V-based applications or fundamental physics investigations.

At the end of the 60ies the first nanostructures caught very rapidly increasing interest of the community. Dingle and coauthors fabricated the first “particle-in-a-box” structure, which is called today a “quantum well” or a “two-dimensional structure”. The fundamental band gap of a thin layer of a narrow-band-gap material, with a thickness below the de Broglie wavelength of a charge carrier, inserted between two barriers of larger-gap materials, was discovered to be thickness dependent, thus confirming the theoretical prediction. The emission wavelength of a laser based on quantum wells is consequently tunable via the thickness of the active layer. This discovery marks the advent of modern nanostructure physics. Soon later in the 70ties and 80ties, research moved to structures of still lower dimensionality, like one-dimensional and zero-dimensional structures, quantum wires and quantum dots (QDs). Efficient technologies for easy fabrication of defect-free nanostructures were missing, however, and the interest faded away until the beginning of the 90ties. Then the Stranski-Krastanow mode of self-organized growth of strained zero-dimensional nanostructures was discovered [1], theoretically founded by modern theory of surface physics and demonstrated to present the basis of active layers for e.g. lasers with lower threshold current density than ever thought of [2]. Surprisingly, two paradigms of modern semiconductor physics had to be given up at the same time by these discoveries: the “lattice match paradigm for heterostructures” and the “fabrication paradigm” that lithography based method must be employed to create quantum wires and QDs. A minimum amount of strain induced by lattice mismatch of the heterostructures is the driving source for QD formation. Zero-dimensional structures, from the point of view of their electronic properties, do not resemble any more classical semiconductors with their continuous dispersion of energy as a function of momentum. They behave like giant hydrogen atoms in a dielectric cage and show a very simple twofold degenerate energy level system [2] thus presenting a potential source of qubits and entangled photons.

In the 21st century, the hallmarks of modern solid state physics, far beyond just semiconductors, are design, fabrication, study and applications of the now existing great variety of nanostructures. Among them, quantum rings, which are the subject of the present book, take an outstanding place, because they are not simply zero-dimensional coherent clusters of atoms or molecules on a surface. Quantum rings combine sizes at the nanoscale with a non-trivial topology: doubly-connectedness of a ring or even more complicated topological properties like one-sidedness of a Möbius strip. This combination leads again to the occurrence of unique physical properties, in particular, persistent currents. Quantum rings present a unique playground for quantum mechanical paradigms. Their physical properties are designed by controlling the geometry of a ring and the magnetic flux threading it, as well as by creating assemblies of quantum rings.

The present book gives an exhaustive and clear overview of this vigorously developing field, starting with a comprehensive pedagogical introduction of the fundamentals, via a profound presentation of the key technologies for their fabrication, characterization tools, discoveries and findings, to a discussion of the most recent

advancements and current research activities. The style of the book is highly motivating for both experienced and young scientists: it finally leads a reader straightforwardly towards still open problems in this fascinating field.

The book is written by a group of the world's leading scientists of this field, who have provided fundamental contributions to the fabrication, characterization and theoretical analysis of quantum rings. Hence a reader receives a unique access to their "scientific laboratory", in particular, about state-of-the art methods of growth (MBE, droplet epitaxy, lithographic patterning, ...), characterization (Scanning-probe imaging like STM, SEM, XSTM, ...) and theoretical analysis of nanostructures and metamaterials.

Based on their unprecedented tunability, quantum rings are highly prospective as elemental base for various applications: photonic detectors and sources, including single-photon emitters, nanoflash memories, qubits for spintronic quantum computing, magnetic random access memory, recording medium and other spintronic devices ... The book contains road maps for the implementation of quantum rings into such real-world devices.

This book will be the required reading for all those who are active in nanoscience, nanotechnology and the applications of quantum rings.

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Dieter Bimberg

Preface

For the first time in monographic literature, the present book provides a broad panorama of the physics of quantum rings with emphasis on *modern advancements* in theoretical and experimental investigations of semiconductor quantum rings. It is written in a style which makes these issues accessible to theoretical physicists, experimental researchers, and technologists with different levels of experience: from graduate and PhD students to experts. The book is also intended to convey the fascination of quantum rings to specialists in other disciplines: mathematics, chemistry, electronic and optical engineering, and information technologies. Our goal is that this book will succeed in invigorating research interests towards the further development of fundamental insight in and applications of quantum rings.

It starts with an introduction into the fundamental physics of quantum rings as a heuristically unique playground for the quantum-mechanical paradigm and a concise overview of the state-of-the-art in the field, with a particular emphasis on the quantum interference phenomena like the Aharonov-Bohm effect in quantum rings (Chap. 1). The book consists of three main parts, though the borders between them are conventional: Part I. Fabrication, characterization and physical properties, Part II. Aharonov-Bohm effect for excitons and Part III. Theory.

The *first part* represents three advanced methods of fabrication of quantum rings: self-organized growth, droplet epitaxy and lithographic patterning, as well as their characterization based on scanning-probe-microscopy. It opens with Chap. 2 (by Wen Lei and Axel Lorke) and Chap. 3 (by Jorge M. García, Benito Alén, Juan Pedro Silveira and Daniel Granados) representing fundamentals of the self-organized growth and optical properties of semiconductor quantum rings. In Chap. 4 (by myself, Vladimir N. Gladilin, Jozef T. Devreese and Paul M. Koenraad) we discuss how the modern characterization of self-assembled InGaAs/GaAs quantum rings using X-STM has allowed for a development of an adequate model of their shape, which quantitatively explains the Aharonov-Bohm effect observed in the magnetization. Self-organized formation of highly distinct GaSb/GaAs quantum-ring structures and their X-STM characterization are presented in Chap. 6 (by Andrea Lenz and Holger Eisele).

Scanning-probe electronic imaging of lithographically patterned quantum rings, which is discussed in Chap. 5 (by Frederico R. Martins, Hermann Sellier, Marco G. Pala, Benoit Hackens, Vincent Bayot and Serge Huant), can access to the intimate properties of buried electronic systems. Another promising way of controllable self-assembled fabrication of quantum rings—by droplet epitaxy—is overviewed in Chap. 7 (by Jiang Wu and Zhiming M. Wang) with emphasis on ordered arrays and in Chap. 8 (by Stefano Sanguinetti, Takaaki Mano and Takashi Kuroda), where the focus is on semiconductor quantum-ring complexes.

The *second part* deals with the Aharonov-Bohm effect for multi-electron systems, in particular, for excitons and plasmons. In Chap. 9, Alexander V. Chaplik and Vadim M. Kovalev review theoretical investigations on novel versions of the Aharonov-Bohm effect in quantum rings, including that for electronic Wigner molecules, polarized neutral and charged excitons, and polarons, as well as its manifestations in the longitudinal magnetoresistance. Also, the role of the spin-orbit interaction in the electronic properties of quantum rings is revealed. Theory meets experiment on the Aharonov-Bohm effect for neutral excitons in quantum rings in Chap. 10 (by Marcio D. Teodoro, Vivaldo L. Campo, Jr., Victor Lopez-Richard, Euclydes Marega, Jr., Gilmar E. Marques and Gregory J. Salamo). Remarkably robust optical Aharonov-Bohm effect occurs in type-II quantum dots presented in Chap. 11 (by Ian R. Sellers, Igor L. Kuskovsky, Alexander O. Govorov and Bruce D. McCombe). Chapter 12 (by Fei Ding, Bin Li, François M. Peeters, Val Zwiller, Armando Rastelli and Oliver G. Schmidt) describes the observation and manipulation of Aharonov-Bohm-type oscillations in a single quantum ring.

The *third part* represents advancements in theory of quantum rings. The effects of a tensile-strained insertion layer on strain and the electronic structure of quantum rings are analyzed in Chap. 13 (by Pilkyung Moon, Euijoon Yoon, Won Jun Choi, Jae Dong Lee and Jean-Pierre Leburton) using the model advanced in Chap. 4. The basic approaches to theoretical modeling of electronic and optical properties of semiconductor quantum rings are overviewed by Oliver Marquardt in Chap. 14; it can also serve as a tutorial for students. A survey on Coulomb interaction in finite-width quantum rings is provided in Chap. 15 (by Benjamin Baxevanis and Daniela Pfannkuche). Booming studies on general topological aspects of quantum rings are illuminated by Benny Lassen, Morten Willatzen and Jens Gravesen in Chap. 16 on differential-geometry methods applied to rings and Möbius nanostructures. In Chap. 17, Carlos Segarra, Josep Planelles and Juan I. Climente discuss effects of hole mixing in semiconductor quantum rings and show that the strong strain potential may compete against the band-offset potential in quantum rings. Engineering of electron states and spin relaxation in quantum rings and quantum dot-ring nanostructures is reviewed in Chap. 18 (by Marcin Kurpas, Elzbieta Zipper and Maciej M. Maška).

The *main message* of the present book is that the front-line methods of fabrication and characterization of quantum rings together with the sophisticated cutting-edge theoretical research have allowed for accumulation of a significant thesaurus of fundamental information on their behavior. This highly diversified knowledge underpins numerous suggestions for prospective applications of quantum rings as a

highly tunable elemental base for future device design and optimization, in particular, in optoelectronics and spintronics, magnetic memory devices, photonic sources and detectors, and information storage and processing.

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With profound gratitude I keep the memory of my parents, who nurtured my aspiration to comprehend the world. Special thanks are due to my wife and children for their understanding and patience in the course of my work on this book.

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