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Statistical Models for Proportions and Probabilities

 Springer

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Preface

Most elementary statistics books discuss inference for proportions and probabilities, and the primary readership for this monograph is the student of statistics, either at an advanced undergraduate or graduate level. As some of the recommended so-called “large-sample” rules in textbooks have been found to be inappropriate, this monograph endeavors to provide more up-to-date information on these topics. I have also included a number of related topics not generally found in textbooks. The emphasis is on model building and the estimation of parameters from the models.

It is assumed that the reader has a background in statistical theory and inference and is familiar with standard univariate and multivariate distributions, including conditional distributions. This monograph may also be helpful for the statistics practitioner who is involved with statistical consulting in this area, particularly with regard to inference for one and two proportions or probabilities.

[Chapter 1](#) looks at the difference between a proportion and probability. It focuses on a proportion leading to the Hypergeometric model and its Binomial approximation, along with inference for the proportion. Inverse sampling is also considered. [Chapter 2](#) focuses on estimating a probability and considers the Binomial distribution in detail as well as inverse sampling. Exact and approximate inferences for a probability are considered. In [Chap. 3](#), the main focus is on comparing two proportions or two probabilities and related quantities such as the relative risk and the odds ratio from the same or different populations using the Multi-hypergeometric or Multinomial distributions. Simultaneous confidence intervals for several parameters are also considered. The Multinomial distribution is the basis for a number of hypothesis and goodness of fit tests, and these are discussed in [Chap. 4](#) with particular attention given to 2×2 tables and matched data. In [Chap. 5](#), we look briefly at two logarithmic models for discrete data, namely the log linear and the logistic models.

I would like to thank two reviewers for their very helpful comments on a previous draft.

Auckland, New Zealand, June 2012

George A. F. Seber

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