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Method of Guiding Functions in Problems of Nonlinear Analysis

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Contents

Introduction

The method of guiding functions (MGF) was originally developed by M.A. Krasnosel'skii and A.I. Perov as one of the tools for solving problems of periodic oscillations in nonlinear systems (see, e.g., [90, 91, 95, 120]). Being geometrically clear and simple to use in applications, it became one of the most powerful and effective instruments for dealing with periodic problems. In the subsequent years it was generalized and extended in various directions. Important aspects of the theory and applications of the MGF were investigated in the works [5, 19–21, 87, 88, 92, 111, 125] and many others. Notice, in particular, that the MGF was extended to differential inclusions and control systems by E.A. Gango and A.I. Povolotskii [63], Yu.G. Borisovich, B.D. Gelman, A.D. Myshkis, and V.V. Obukhovskii [25], and L. Górniewicz and S. Plaskacz (see [64, 67]). In order to study the periodic problem for functional differential equations, A. Fonda [55] introduced the notion of an integral guiding function. The method of integral guiding functions was developed and used in works of S. Kornev and V. Obukhovskii (see [83, 84]). Starting from the works of R.E. Gaines and J.L. Mawhin (see [62, 111]), the notion of a bounding function, closely related to the concept of a guiding function, was systematically used for the study of various boundary value problems by J. Andres, L. Malaguti, V. Taddei, and other researchers (see [5–10, 15]).

In many problems of nonlinear oscillations arises the necessity to use guiding functions which are non-smooth. In particular, such situation appears when different smooth guiding functions are defined in various domains of the phase space of the system. To study these types of problems, F.S. De Blasi, L. Górniewicz, and G. Pianigiani [37] introduced the notion of a non-smooth guiding potential for differential inclusions with convex-valued and non-convex-valued right-hand sides. This notion was extended and developed by G. Gabor and R. Pietkun in [61], S. Kornev and V. Obukhovskii in [82, 84, 85] and M. Filippakis, L. Gasin'ski, and N.S. Papageorgiou in [53] and, by using the methods of non-smooth analysis, applied to various oscillation problems in systems governed by differential inclusions.

It is worth noting that, beginning from the pioneering works, the MGF was applied almost exclusively to objects in finite-dimensional spaces. Only recently, with the use of approximative schemes, the MGF was extended to systems governed by differential inclusions in infinite-dimensional Hilbert spaces in the papers of N.V. Loi, V. Obukhovskii, and P. Zecca [100, 108, 109].

Meantime, it was found that the MGF can be not only useful to justify the existence of oscillations but also successfully applied to the study of the qualitative behavior of branches of periodic solutions. Using the generalized form of guiding functions W. Kryszewski considered in [96] the global bifurcation problem for periodic solutions of first-order differential inclusions in finite-dimensional spaces. In a cycle of works [101, 102, 104, 107, 109, 115], the systematic investigation and applications of various modifications of the MGF to the global bifurcation problem for several types of inclusions (differential and functional differential inclusions, operator inclusions) were carried out.

Recently, two new branches of the applications of the MGF arose. The first one is the evaluation of an oriented coincidence index for inclusions containing a nonlinear Fredholm operator of zero index through the index of guiding functions. In the work [107] the MGF was used to calculate the oriented coincidence index for a class of feedback control systems that allowed to obtain the existence result for periodic trajectories of such systems. The second approach is the application of the MGF to the study of boundary value problems for second-order differential inclusions (see [105]).

In our opinion all these directions demonstrate that the MGF plays a remarkable and important role in problems of contemporary nonlinear analysis. Our target is to reflect these branches in this monograph.

The plan of the book is as follows.

In order to make the book self-contained, we devote the first chapter to a detailed description of the fundamental, general properties of multimaps and some topological characteristics (topological degree and coincidence degree) that will be used in the next chapters. In particular, we discuss different types of continuity for multimaps and various operations on multimaps. We describe main properties of measurable multifunctions and superposition multioperator which is routinely used when dealing with differential inclusions. We devote particular attention to the problem of the existence of single-valued approximation for multimaps and present approximation properties of multimaps. These properties allow to give the construction and to describe the main features of the topological degree for a wide class of multimaps. The last part of the first chapter contains the description of the coincidence degree theory for pairs consisting of zero-index linear Fredholm operators and multimaps.

In the second chapter, we present the MGF and its modifications for solving various problems for differential inclusions in finite-dimensional spaces. Starting from “classical” applications to a periodic problem, we consider non-smooth and integral guiding functions. We study generalized periodic problems (including known anti-periodic problem) and consider its applications to differential games. The last part of the chapter is devoted to applications of the MGF to global bifurcation problems. After presenting the abstract result, we consider the global bifurcation of periodic solutions and describe the applications to equations with discontinuities, ordinary and functional differential inclusions, and feedback control systems.

The third chapter contains the extension of the MGF to the case of differential inclusions in infinite-dimensional Hilbert spaces. To this aim we use the notion of approximate solvability for operator inclusions (this notion is closely related to the notion of A-proper operator developed by F.E. Browder and W.V Petryshyn [30]). Some sufficient conditions for the approximate solvability of inclusions are given. We apply our results to study differential and functional differential inclusions and feedback control systems and to investigate global bifurcation problem for differential inclusions in Hilbert spaces.

In the fourth chapter, by using the MGF, we obtain existence theorems for a boundary value problem for second-order differential inclusions in finite-dimensional and infinite-dimensional Hilbert spaces. It is shown that the abstract result can be applied to study equations with discontinuous nonlinearities, boundary value problems for differential inclusions, and feedback control systems and to the problem of the motion of a particle.

The last chapter is devoted to the nonlinear Fredholm inclusions. After describing the construction of an oriented coincidence index, we present an approach to calculate it through the use of the index of an appropriate guiding function. Furthermore, we prove an abstract global bifurcation theorem for inclusions containing nonlinear Fredholm operators of index zero. We also show how the MGF can be applied to bifurcation problem for feedback control systems with nonlinear Fredholm operators.

Having explained the title and the plan of the book, we would like to stress again that our main goal is to give a self-contained introduction to the method of guiding functions which allows to study effectively various problems arising in the theory of differential inclusions and control systems in finite-dimensional and Hilbert spaces. The book contains all related results of the authors presented in the works [25, 82–86, 99–109, 115, 117].