

# Quantum Science and Technology

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# **Quantum Science and Technology**

## **Aims and Scope**

The book series Quantum Science and Technology is dedicated to one of today's most active and rapidly expanding fields of research and development. In particular, the series will be a showcase for the growing number of experimental implementations and practical applications of quantum systems. These will include, but are not restricted to: quantum information processing, quantum computing, and quantum simulation; quantum communication and quantum cryptography; entanglement and other quantum resources; quantum interfaces and hybrid quantum systems; quantum memories and quantum repeaters; measurement-based quantum control and quantum feedback; quantum nanomechanics, quantum optomechanics and quantum transducers; quantum sensing and quantum metrology; as well as quantum effects in biology. Last but not least, the series will include books on the theoretical and mathematical questions relevant to designing and understanding these systems and devices, as well as foundational issues concerning the quantum phenomena themselves. Written and edited by leading experts, the treatments will be designed for graduate students and other researchers already working in, or intending to enter the field of quantum science and technology.

Kia Manouchehri • Jingbo Wang

# Physical Implementation of Quantum Walks

 Springer

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# Preface

Random walks have been employed in virtually every science related discipline to model everyday phenomena such as biochemical reaction pathways and DNA synapsis (Sessionsa et al. 1997), the genomic distance from DNA sequence location in cell nuclei (van den Engh et al. 1992), optimal search strategies for hidden targets such as animals' foraging (Bénichou et al. 2005), diffusion and mobility in materials (Trautt et al. 2006), the trail of a particle undergoing brownian motion (Stewart 2001) as well as exchange rate forecast (Kilian and Taylor 2003). They have also found algorithmic applications, for example, in solving differential equations (Hoshino and Ichida 1971), quantum monte carlo for solving the many body Schrödinger equation (Ceperley and Alder 1986), optimization (Berg 1993), clustering and classification (Schöll and Schöll-Paschingerb 2003), fractal theory (Anteneodo and Morgado 2007) or even estimating the relative sizes of Google, MSN and Yahoo search engines (Bar-Yossef and Gurevich 2006).

Whilst the so called *classical* random walks have been successfully utilized in such a diverse range of applications, *quantum* random walks are expected to provide us with a new paradigm for solving many practical problems more efficiently (Aharonov et al. 1993; Knight et al. 2003b). In fact quantum walks have already inspired efficient algorithms with applications in connectivity and graph theory (Kempe 2003b; Douglas and Wang 2008), as well as quantum search and element distinctness (Shenvi et al. 2003; Childs and Goldstone 2004b), due to their non-intuitive and markedly different properties, including faster *mixing* and *hitting* times. And more recently, some quantum walk processes are shown to be capable of acting as universal computational primitives (Childs 2009).

The emerging prospects for a new generation of quantum algorithms inspired by quantum walks have naturally fuelled a second area of research: developing the physical "hardware" that is capable of performing a quantum walk in the laboratory. As well as being experimentally viable, such a physical implementation is expected to provide a natural mechanism by which it can be scaled up, enabling it to deal with modestly large practical problems. Moreover, while purpose built systems for implementing specific quantum walk algorithms may be more straightforward to

design, when considering a hypothetical problem formulated in terms of a particular type of graph, developing a problem-independent implementation scheme that is not limited to specific connectivity criteria is highly desirable.

Over the past decade there have been several proposals for implementing quantum walks, utilizing a variety of quantum, classical and hybrid systems including Nuclear Magnetic Resonance, cavity QED, ion traps, optical traps, optical networks and quantum dots. Of these, a vast majority provide constructive insights into the elements of a successful physical implementation, though in themselves are unsuitable as a practical scheme. Other proposals have considered the notions of feasibility, scalability and generality of application, but only to a limited extent. Nonetheless, while building a large scale quantum walk machine remains a considerable challenge in the foreseeable future, “proof of principle” implementations have already been experimentally demonstrated for a number of proposals.

We begin this book with a brief overview of quantum walk theory, including a description of the two main classes of walks, namely, *continuous-time* quantum walks and *discrete-time* or *coined* quantum walks, as well as their properties and applications; areas which have already received substantial treatment in other reviews including those of Kempe (2003b), Ambainis (2003), Kendon (2007), and Venegas-Andraca (2012). The main focus of this book however will be the *physical implementations* of quantum walks examined in the subsequent chapter, where we present a comprehensive survey of numerous implementation schemes to date. The tremendous diversity of approaches in these proposals has necessitated references to a wide array of underlying physical phenomena, particularly in relation to the field of quantum optics. Therefore, to assist the reader while maintaining continuity throughout the book, a considerable body of supplementary material and background theory has been included in the appendices.

In carrying out the original research described in this book, we have greatly benefited from valuable and stimulating discussions with many pioneering theorists and experimentalists in this field, in particular Gerard Milburn, Jason Twamley, Gavin Brennen, Peter Rohde, Jeremy O’Brien, Paolo Metaloni, Dieter Meschede, Jonathan Matthews, Andreas Schreiber, Norio Konno, Etsuo Segawa, Yutaka Shikano, Armando Perez, and Miklos Santha, for which we are truly grateful. We would also like to thank Zhijian Li, Michael Delanty, and Stefan Danilishin for their careful and critical proofreading of the manuscript, although any errors or omissions remain solely our responsibility. Special mention should be made of a number of students, most notably Brendan Douglas, Scott Berry, and Thomas Loke, who contributed towards our original research work presented here. The University of Western Australia has provided a rich intellectual environment that led to the completion of this book. The support and encouragement of Ian McArthur and Jim Williams at the School of Physics, as well as Angela Lahee (the editor) and Priya Balamurugan (the production editor) of Springer are also sincerely acknowledged.

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