An Introduction to Inverse Problems with Applications

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Springer

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This book is dedicated to

my love Maysa, our beloved children Daniel and Cecilia,
and my dear parents Maria Luiz and Francisco *(in memoriam)*

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my beloved ones, Gilsineida, Lucas and Luísa,
and to my dear parents Antônio *(in memoriam)* and Jarleide

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Foreword

For the benefit of the general readership, it might be a good idea to first explain the difference between "inverse problems" versus "analysis problems" using simple general terms as defined by Professor Shiro Kubo of Osaka University in his book published in the early 1990s.

Analysis problems are well-posed problems concerned with finding distribution(s) of certain variable(s) in a domain of a given size and shape that can be multiply connected and time-dependent. Results of the analysis problems also depend on boundary and/or initial conditions given at every point of a boundary of the domain. Properties of the media filling the domain must also be given in addition to strengths and locations/distribution of any possible sources/sinks. Finally, equation(s) governing the field variable(s) must be given. If all of these pieces of information are given, then the problem of finding the field distribution of variable(s) is a well-posed problem that can be solved using appropriate numerical integration algorithms.

However, if at least one of these pieces of information is missing, such under-specified problems become inverse problems of determining the missing piece of information in addition to simultaneously solving the original analysis problem. To enable ourselves to accomplish this complex task, we must be given an additional piece of information (typically, a part of the solution of the corresponding analysis problem) which makes the inverse problem an over-specified or ill-posed problem.

For example, when given size and shape of an isotropic plate and either Neumann or Dirichlet boundary conditions at every point along the edges of the plate, steady state heat conduction in the plate will be governed by the LaPlace's equation for temperature. This would be a classical well-posed analysis problem. However, if boundary conditions are not given on one boundary of this plate, the problem of finding temperature distribution in the plate becomes under-specified and cannot be solved. This problem will become solvable if we provide both Dirichlet and Neuman boundary conditions simultaneously on at least some parts of the plate's boundaries which will make this an over-specified or ill-posed inverse problem of determining the missing boundary conditions and simultaneously determining the distribution of temperature throughout the plate.

Inverse problems have traditionally been considered mathematically challenging problems and have consequently been studied predominantly by mathematicians. Since there are many practical inverse problems in a variety of disciplines that require mathematical tools for their solution, it is scientists and engineers that have been developing many of these methods recently out of necessity to obtain practical results. Consequently, an initially wide gap between scientists and engineers versus applied mathematicians has been steadily narrowing as both communities have realized that they have many things to learn from each other.

This book is a welcome and unique publication that uses a common language to blend the rigour of the applied mathematics world and the reality of a research scientist's or an engineer's world. Thus, it should appeal to everyone who has the basic knowledge of differential equations and at least a rudimentary understanding of basic mathematical models used in field theory and general continuum mechanics and transport processes. Specifically, applied mathematicians will be able to find here physical relevance for some of their theoretical work and learn to appreciate the importance of developing understandable, easy-to-use, easy to adapt and reliable algorithms for the solution of different classes of inverse problems. At the same time, research scientists and engineers will be able to learn from this book that some of the methods and formulations that they have been using in the past are prone to problems of non-uniqueness of the results and that accuracy of many of the practical methods could easily become a real issue when solving inverse problems.

Actually, this book could be used not only as a valuable reference book, but also as a textbook for students in the fields of applied mathematics, engineering and exact sciences. Besides its simple language, this book is easy to comprehend also because it contains a number of illustrative examples and exercises demonstrating each of the major concepts and algorithms.

For example, basic concepts of regularization of ill-posed problems are very nicely explained and demonstrated so that even a complete novice to this field can understand and apply them. Formulations and applications in image processing and thermal fields presented in this book have direct practical applications and add significantly to the more complete understanding of the general problematics of inverse problems governed by elliptic and parabolic partial differential equations. Inverse scattering problems have not been covered in this book as this field can easily fill a separate book.

Many formulations for the solution of inverse problems used to be very discipline specific and even problem specific. Thus, despite their mathematical elegance and solution efficiency and accuracy, most of the classical inverse problems solution methods had severe limitations concerning their fields of applicability. Furthermore, most of these methods used to be highly mathematical, thus requiring highly mathematical education on the part of users.

Since industry requires fast and simple algorithms for the solution of a wide variety of inverse problems, this implies a growing need for users that do not have a very high degree of mathematical education. Consequently, many of the currently used general algorithms for the solution of inverse problems eventually result in some sort of a functional that needs to be minimized. This has been recognized by the authors of this book which have therefore included some of the most popular minimization algorithms in this text.

Hence, this book provides a closed loop on how to formulate an inverse problem, how to choose an appropriate algorithm for its solution, and how to perform the solution procedure.

I recommend this book highly to those that are learning about inverse problems as well as to those that think that they know everything about such problems. Both entities will be pleasantly surprised with the ease that concepts, formulations and solution algorithms are explained in this book.

George S. Dulikravich

Miami, Florida

June 2011

Preface

Archimedes, is this crown made of gold?
King Hiero II of Syracuse[a], *circa* 250 BCE.

(...) to find a shape of a bell by means of the sounds which it is capable of sending out.
Sir A. Schuster[b], 1882.

Can one hear the shape of a drum?
Marc Kac[c], 1966.

[a] Hiero II (308 BCE - 215 BCE).
[b] Sir A. Schuster (1851-1934).
[c] Marc Kac (1914-1984).

On inverse problems

Perhaps the most famous inverse problem for the mathematical community is: *Can one hear the shape of a drum?* [40, 65]. That is: *Is one able to figure out the shape of a drum based on the sound it emits?* The corresponding direct problem is to determine the sound emitted by a drum of known shape. The solution to the direct problem is long known, but the solution to the inverse problem eluded the scientific community for a long time. It was found to be negative: there are two drums, different in shape, that emit the same sound, see [36]. Several other mathematical aspects concerning the resolution of inverse problems have been investigated in recent years.

Parallel to that, a large number of significant inverse problem methodology applications were developed in engineering, medicine, geophysics and astrophysics, as well as in several other branches of science and technology.

Why? Because inverse problems is an interdisciplinary area that matches the mathematical model of a problem to its experimental data. Or, given a bunch of numbers, data, in a data driven research, looks for a mathematical model. It is an interface between theory and practice!

About this book

The general purpose of this book is to introduce certain key ideas on inverse problems and discuss some meaningful applications. With this approach, we hope to be able to stimulate the reader to study inverse problems and to use them in practical situations.

The book is divided, though not in sequence, in two main parts, one of a more mathematical nature, and the other more linked to applications. It adopts an elementary approach to the mathematical analysis of inverse problems and develops a general methodology for the solution of real inverse problems. Further, it discusses a series of applications of this methodology, ranging from image processing applied to medicine, to problems of radiation applied to tomography, and onto problems of conductive heat transfer used in the design of thermal devices. The choice of applications reflect the acquaintance of the authors.

In order to make the book of a manageable size and suitable for a larger audience, we opted to make the presentation of mathematical concepts in the context of linear, finite dimensional, inverse problems. In this setting, key issues can be handled with mathematical care, in a rather “pedestrian” and easy way because the problem is linear and finite dimensional, requiring only acquaintance with basic ideas from linear algebra. Geometrical ideas, which are a key to generalization, are emphasized.

Some of the applications considered, however, involve problems which are neither finite dimensional nor linear. The treatment then is numerical, and the ideas from the first part of the book are used as guidelines, through extrapolation. This is possible, in part, because, to simplify, this book deals, several times, with *least squares methods.* Although the subjects in this book are intricate, the chapters can be read, somewhat, independently of one another. This is because of the intended redundancy, employed for pedagogical reasons, like in an old teaching’s tradition: attention, association and repetition. To make it easier to peruse the book, a description of the book’s content, chapter by chapter, is given on pages 4-6.

The pre-requisites to read this book are calculus of several variables and linear algebra. Nonetheless, a few concepts and results from linear algebra and calculus are reviewed in the appendix, in order to make the book reasonably self-contained. Even though knowledge of differential equations is necessary to understand some parts, basic concepts on this subject are not supplied or reviewed. Knowledge of numerical methods might be useful for reading certain sections. We included exercises at the end of each chapter, many of them guided, to make it easier for readers to grasp, and extend the concepts presented.

We believe that this book can be read, with some interest and to their profit, by upper division undergraduate students and beginning graduate students in applied mathematics, physics, engineering, and biology. Since it also includes some thoughts on mathematical modeling, which are in the back of the minds of researchers, but are not usually spelled out, this book may interest them too.

Some remarks

Practical matters: we use *emphasised expressions* to signal a group of words with a specific meaning when they are being defined either explicit or implicitly. The end of an example, a proof, or an argument is indicated by a small black square. ■

The authors will be happy to receive any comments and/or suggestions on this book.

This text reflects the partnership between a mathematician and an engineer. It is the result of a cross fertilization, in full collaboration, that greatly enthuses us and that, we hope, encourages others to break the usual barriers and pursue similar partnerships.

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July 2012

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A previous version of this book was published originally in Portuguese, but additional material has been included here. We recognize and appreciate the careful initial translation into English of the previous book by Mr. Rodrigo Morante.

During all these years we have been working on inverse problems, we benefited from discussions with several colleagues and students, which greatly helped us to understand better the issues involved on their solutions and we thank them all.

We want to acknowledge our colleagues and staff of Instituto Politécnico from the Rio de Janeiro State University, in Nova Friburgo, for constructing and mantaining such a challenging academic environment in the inner *Mata Atlântica*, *Serra do Mar*, a remaining part of the original rain forest in the State of Rio de Janeiro, a hundred miles northeast from the city of Rio de Janeiro.

The campus of Instituto Politécnico was severely affected by the torrential rains and devastating landslides that occurred on january 11, 2011 in the city of Nova Friburgo[1]. As a result, we were *campus-less* by more than a year and just recently, on march 2012, moved to a new campus. During this *annus horribilis* our Instituto was maintained ‘alive’ by brave colleagues and staff that take on the mission of education heartily. We again thank them all.

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[1] NY Times http://www.nytimes.com/2011/01/14/world/americas/14brazil.html

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We register our thanks to Rui Ribeiro de Azevedo for helping us with the book cover and to Maysa Sacramento de Magalhães who took the photo in the book cover.

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