Texts in Computational Science and Engineering

9

Editors

Timothy J. Barth
Michael Griebel
David E. Keyes
Risto M. Nieminen
Dirk Roose
Tamar Schlick

For further volumes:
http://www.springer.com/series/5151

Michael Bader

Space-Filling Curves

An Introduction with Applications in Scientific Computing

Springer

Michael Bader
Department of Informatics
Technische Universität München
Germany

ISSN 1611-0994
ISBN 978-3-642-31045-4 ISBN 978-3-642-31046-1 (eBook)
DOI 10.1007/978-3-642-31046-1
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2012949128

Mathematics Subject Classification (2010): 68W01, 14H50, 65Y05, 65Y99, 68U99, 90C99

Printed on acid-free paper

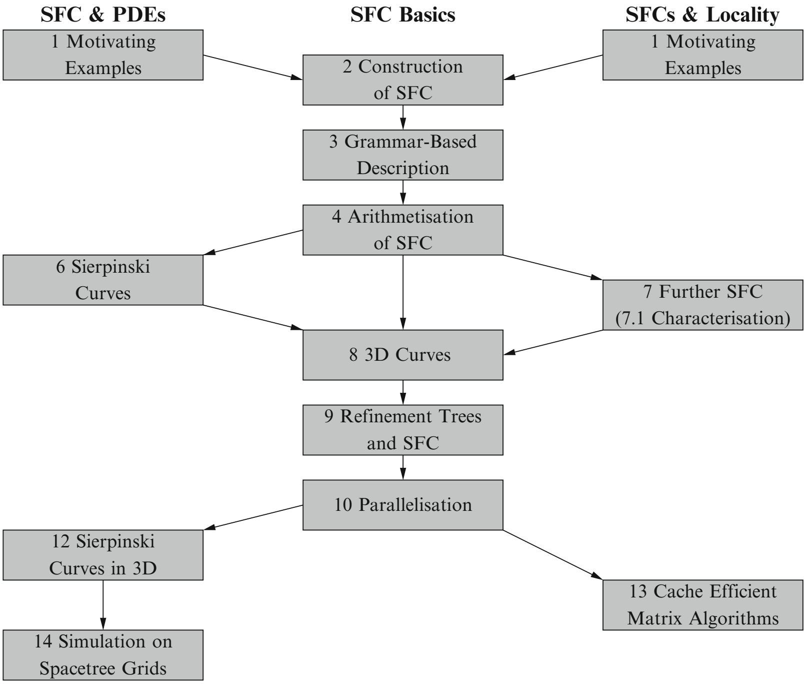
Springer is part of Springer Science+Business Media (www.springer.com)

Preface

Space-filling curves started their "lives" as mathematical curiosities, at the end of the nineteenth century. The idea that a one-dimensional curve may completely cover an area or a volume was, at that time, completely novel and counter-intuitive. Together with their relatives – such as Koch curves, the Cantor Set, and similar constructions – space-filling curves contradicted the then existing notion of a curve and of continuity and differentiability of functions (being continuous everywhere, but nowhere differentiable), and demanded new concepts in set theory and dimensionality. As a result, space-filling curves became almost notorious as "topological monsters", and were studied by a good number of highly influential mathematicians, such as Peano, Hilbert, Lebesgue, Sierpinski, and others. The book of Hans Sagan [233] provides an excellent introduction and overview of these mathematical aspects of space-filling curves, as well as on their history.

The recursive and self-similar construction of space-filling curves leads to important locality properties – to put it in a nutshell, a space-filling-curve mapping will ensure that close-by parameters will be mapped to neighbouring points in the target set and – at least to a good extent – vice versa. It turned out that these properties are highly useful in the computational sciences. Algorithms based on space-filling curves can thus be used to construct spatial partitions to distribute a problem to different processors of a parallel computer, to improve the memory access behaviour of algorithms, or in general to find efficient data structures for multidimensional data. Most of the time, space-filling curves are not necessarily the best algorithm for the job where they are used – however, as Bartholdi and Platzman [32] put it, they are like "a good pocketknife", being "simple and widely applicable", and able to provide a convenient and satisfactory solution with comparably small effort in implementation and computation.

The aim of this book is therefore to give an introduction to the *algorithmics* of space-filling curves – to the various ways of describing them, and how these different description techniques lead to algorithms for different computational tasks. The focus will be on algorithms and applications in scientific computing, with a certain preference on mesh-based methods for partial differential equations.



How to Read This Book

The present book will hopefully serve multiple purposes – as a monograph on space-filling curves; as a reference to look up specific aspects, algorithms, or techniques to describe a specific curve; or as a textbook to be used as supplementary material for a course related to scientific computing, or even for a series of lectures that is dedicated to space-filling curves in particular. As a consequence, the sequential order of chapters I chose for this book will necessarily not be able to match the needs of every reader or lecturer. You are thus encouraged to read through this book in a non-sequential way, skip chapters or sections, do detours to later chapters, or similar. Some suggestions for selecting chapters and placing a different focus during a course ("just the basics" vs. techniques for partial differential equations vs. special focus on locality properties) are given in the figure above. In addition, every chapter will end with a box called "What's next?" that will give some suggestions on where to read on.

Additional material (solution to exercises, code examples, links to other material, errata if necessary) will be published on the website

`www.space-filling-curves.org`.

Acknowledgements

The topic of space-filling curves has been a part of my research and teaching work for the last 8 years, at least, and numerous colleagues and fellow researchers provided valuable input and ideas. I would especially like to thank all my colleagues at Technische Universität München and at Universität Stuttgart – while I cannot list all of them, I want to express my special gratitude to those who have been my colleagues for the longest time: Hans Bungartz, Miriam Mehl, Tobias Neckel, Tobias Weinzierl, and Stefan Zimmer. Amongst all colleagues from other research groups, I would like to give thanks and tribute to Jörn Behrens, Herman Haverkort, Bill Mitchell, and Gerhard Zumbusch, who provided lots of inspiration for this book – especially for the chapters on Sierpinski curves and on locality properties.

Finally, and most of all, I want to thank Christoph Zenger, my former academic supervisor and mentor. In 2003, we initiated a joint lecture on *Algorithms in Scientific Computing*, with space-filling curves being one of three major topics. It was the first course, in which I was solely responsible for the content of a lecture, it has been running (with certain topical updates, of course) in this form for 8 years, and it was the starting-point and foundation for this book project.

Munich April 23, 2012 Michael Bader

Contents