

Basic Concepts of String Theory

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Ralph Blumenhagen
Dieter Lüst
Stefan Theisen

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Ralph Blumenhagen
Werner-Heisenberg-Institut
Max-Planck-Institut für Physik
München
Germany

Stefan Theisen
Albert-Einstein-Institut
Max-Planck-Institut für Gravitationsphysik
Golm
Germany

Dieter Lüst
Ludwig-Maximilians Universität München
Arnold-Sommerfeld Zentrum für
Theoretische Physik
München
Germany

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Preface

This is a revision of Vol. 346 of Springer Lecture Notes in Physics written by D. Lüst and S. Theisen in 1989. The notes have long been out of print, but over the years we had continuous positive feedback from students, who found the book helpful in their attempt to enter string theory. To make the book useful for a new generation of string theorists required a revision and a substantial extension. Unfortunately this means that it became intimidatingly voluminous.

The purpose of this new edition is the same as of the old one: to prepare the reader for research in string theory. It is not a compendium of results but is intended to be a textbook in the sense that, at least in most parts, the reader is not referred to the original literature for derivations. We try to be pedagogical, avoiding excessive use of phrases such as “it is well known,” “one can show,” which are often frustrating (not only) for the beginner.

Aiming at a pedagogical introduction to a vast subject such as string theory also means that we had to make a selection of topics. Major omissions are black holes in string theory, strings at finite temperature, string cosmology, anomalies in string theory, model building, matrix model description of M-theory, string field theory, to name a few.

We give references at the end of each chapter. We restrict ourselves to some basic papers and reviews from which we have profited and also some additional references which cover material which goes beyond what we could cover. Almost all references are easily available. With few exceptions they are either published in journals, available as preprints on the arXiv or scanned at: http://www-lib.kek.jp/KISS/kiss_preprint.html.

The influence of the classic string monographs by Green, Schwarz, Witten, and Polchinski can be felt throughout most chapters.

München, Germany
München, Germany
Golm, Germany

Ralph Blumenhagen
Dieter Lüst
Stefan Theisen

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