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Explicit Nonlinear Model Predictive Control

Theory and Applications

Springer

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To my parents Assoc. Prof. Dr. Siyka Popova
and Prof. D.Sc. Ivan Grancharov for their
love and support
(Alexandra Grancharova)

Preface

Model predictive control (MPC) has become the accepted methodology to solve complex control problems related to process industries. It allows the design of multi-input multi-output (MIMO) control systems that minimize a certain performance index in the presence of input and output constraints. The Nonlinear Model Predictive Control (NMPC) is an optimization-based method for control which involves the solution at each sampling instant of a finite horizon optimal control problem subject to the *nonlinear* system dynamics and input and output constraints imposed on the system. However, the solution of an on-line *nonlinear* optimization problem is often computationally complex and time consuming and the real-time NMPC implementation is usually limited to slow processes where the sampling time is sufficient to support the computational needs. The on-line computational complexity can be circumvented with an *explicit* approach to NMPC, where an *explicit* approximate representation of the solution is computed using multi-parametric Nonlinear Programming (mp-NLP).

Motivation

The main motivation behind *explicit* MPC is that an *explicit* state feedback law avoids the need for executing a numerical optimization algorithm in real time, and is therefore potentially useful for applications where MPC has not traditionally been used. It has been shown that the feedback solution to MPC problems for constrained *linear* systems has an *explicit* representation as a piecewise linear state feedback defined on a polyhedral partition of the state space. The benefits of an *explicit* solution, in addition to the efficient on-line computations, include also verifiability of the implementation (which is an essential issue in safety-critical applications) and the possibility to design embedded control systems with low software and hardware complexity. For *nonlinear* MPC the prospects of *explicit* solutions are even higher than for *linear* MPC, since the benefits of computational efficiency and verifiability are even more important.

The main reasons to develop methods for *explicit* NMPC can be summarized as follows:

- Dramatical reduction in online computations, since online *nonlinear* numerical optimization is avoided and replaced by piecewise function evaluation. This may lead to significant reduction in the requirements to real-time embedded computer hardware.
- NMPC optimization depends on appropriate initialization in order to avoid local minima, and appropriate formulation of constraints in order to avoid infeasibility. With *explicit* NMPC the validation of initialization procedures and infeasibility handling can be conducted based on a complete and *explicit* solution.
- Significant reduction in online software complexity since the code for piecewise function evaluation is much simpler than a *nonlinear* numerical optimization solver. This may lead to formal software verification being a feasible practical tool.
- Approximate *explicit* solutions with reduced complexity, and with guaranteed levels of sub-optimality, may be computed offline. Formal analysis of performance, sub-optimality and stability may be possible since an *explicit* representation of the controller is known.
- Formulations such as stochastic NMPC and robust NMPC may not lead to increased online computations in an *explicit* NMPC approach, compared to a nominal NMPC formulation, although they will require more offline computations.

Main contributions of the book

This book considers the mp-NLP approaches to *explicit* approximate NMPC of constrained *nonlinear* systems, developed by the authors, as well as their applications to various NMPC problem formulations and several case studies. The proposed mp-NLP methods are based on orthogonal partition of the state space and they are general in sense that they can be applied to solve both convex and non-convex optimization problems. The following types of *nonlinear* systems are considered, resulting in different NMPC problem formulations:

- *Nonlinear* systems described by first-principles models and *nonlinear* systems described by black-box models;
- *Nonlinear* systems with continuous control inputs and *nonlinear* systems with quantized control inputs;
- *Nonlinear* systems without uncertainty and *nonlinear* systems with uncertainties (polyhedral description of uncertainty and stochastic description of uncertainty);
- *Nonlinear* systems, consisting of interconnected *nonlinear* sub-systems.

The proposed mp-NLP approaches to *explicit* solution of various NMPC problems are illustrated with applications to several case studies, which present mathematical models, NMPC formulations, mp-NLP computational results, and closed loop simulations. They are taken from diverse areas such as automotive mechatronics, compressor control, combustion plant control, reactor control, pH maintaining system control, cart and spring system control, and diving computers.

Intended audience

The book is intended to support graduate courses and the study of Ph.D. and advanced M.Sc. students in *nonlinear* control and optimization. Readers should be familiar with the basics of linear model predictive control, numerical optimization methods, and linear and nonlinear control theory. The book could be also useful for academic researchers working in the field of NMPC, as well as researchers from industrial companies, including automotive and aerospace, whose responsibilities include the development of embedded optimal control systems.

Book organization

The book is structured as follows:

- In **Chapter 1**, basic theory and algorithms to find an *explicit* approximate solution of mp-NLP problems, based on orthogonal ($k-d$ tree) partition of the parameter space, are described by considering both the convex and the non-convex case. Procedures and heuristic rules for efficient splitting of a region in the parameter space and for handling the infeasible cases are formulated.
- In **Chapter 2**, the main aspects of formulation of the NMPC optimization problem are considered, which is an essential part of the control design and involves numerous decisions that are important for the control performance, feasibility, stability, and robustness as well as the computational complexity and the numerical challenges of computing the solution.
- In **Chapter 3**, an algorithm for *explicit* NMPC, which locally approximates the mp-NLP problem with a multi-parametric quadratic program is described. The approach is applied to a case study.
- **Chapter 4** considers the design of *explicit* NMPC controllers for several case studies by applying the approximate mp-NLP algorithms, described in Chapter 1. The case studies present mathematical models, NMPC formulations, mp-NLP computational results, and closed loop simulations. They are taken from diverse areas such as automotive mechatronics, compressor control, and diving computers. In this chapter, it is also shown that bounding the approximation error of the *explicit* approximate solution to convex regulation NMPC problems ensures the asymptotic stability of the suboptimal closed-loop system.
- **Chapter 5** presents an approximate multi-parametric Nonlinear Integer Programming (mp-NIP) approach to design *explicit* NMPC controllers for constrained nonlinear systems with quantized control inputs. The approach is applied to two case studies.
- In **Chapter 6**, two approaches to *explicit* min-max NMPC of constrained nonlinear systems in the presence of bounded disturbances and/or parameter uncertainties are considered. The first approach is based on an open-loop min-max NMPC problem statement, while the second approach adopts a closed-loop min-max NMPC formulation. With the latter approach, conditions for guaranteeing the l_2-stability of the closed-loop system are derived. Two case studies are considered.
- In **Chapter 7**, two approaches to *explicit* stochastic NMPC of constrained nonlinear systems in the presence of disturbances and/or parameter uncertainties with known probability distributions are presented. The first approach constructs

explicit approximate NMPC solution for systems, described by stochastic parametric models, while the second approach considers systems, described by Gaussian process models. The approaches are applied to two case studies.

- **Chapter 8** considers an approximate mp-NLP approach to *explicit* solution of output-feedback NMPC problems for constrained nonlinear systems described by neural network NARX models. A dual-mode control strategy is proposed in order to achieve an offset-free closed-loop response in the presence of bounded disturbances and/or model errors. One case study is considered.
- In **Chapter 9**, a suboptimal approach to distributed NMPC for systems consisting of nonlinear subsystems with linearly coupled dynamics, subject to both state and input constraints, is considered. The approach is based entirely on distributed on-line optimization and can be applied to large-scale nonlinear systems. Also, a *semi-explicit* NMPC approach to efficiently solve the distributed NMPC problem for small- and medium-scale systems is proposed. Both distributed NMPC approaches are applied to an example nonlinear system.

Alexandra Grancharova has been the main contributor to Chapters 1 and 4 – 9, and Tor Arne Johansen has been the main contributor to Chapters 2 and 3.

Sofia,
Trondheim,
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Tor Arne Johansen

Contents