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# Intelligent Routines

Solving Mathematical Analysis with Matlab,  
Mathcad, Mathematica and Maple

 Springer

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Dedicated to our families.

”Homines dum docent discunt.”

Seneca, Epistole 7

”Nihil est in intellectu, quod non prius fuerit in sensu.”

John Locke

”Les beaux (grands) esprits se rencontrent.”

Voltaire

”Men should be what they seem,  
Or those that be not, would they might seem none.”

Shakespeare, Othello III. 3

”Science needs a man’s whole life. And even if you had two lives, they would not be enough. It is great passion and strong effort that science demands to men ...”

I. P. Pavlov

VIII

”Speech is external thought, and thought internal speech.”

A. Rivarol

”Nemo dat quod non habet.”

Latin expression

”Scientia nihil aliud est quam veritatis imago.”

Bacon, *Novum Organon*

# Preface

Real Analysis is a discipline of intensive study in many institutions of higher education, because it contains useful concepts and fundamental results in the study of mathematics and physics, of the technical disciplines and geometry.

This book is the first one of the kind that solves mathematical analysis problems with all four related main software Matlab, Mathcad, Mathematica and Maple.

Besides the fundamental theoretical notions, the book contains many exercises, solved both mathematically and by computer, using: Matlab 7.9, Mathcad 14, Mathematica 8 or Maple 15 programming languages.

Due to the diversity of the concepts that the book contains, it is addressed not only to the students of the Engineering or Mathematics faculties but also to the students at the master's and PhD levels, which study Real Analysis, Differential Equations and Computer Science.

The book is divided into nine chapters, which illustrate the application of the mathematical concepts using the computer. The introductory section of each chapter presents concisely, the fundamental concepts and the elements required to solve the problems contained in that chapter. Each chapter finishes with some problems left to be solved by the readers of the book and can be verified for the correctness of their calculations using a specific software such as Matlab, Mathcad, Mathematica or Maple.

The first chapter presents some basic concepts about the theory of sequences and series of numbers.

The second chapter is dedicated to the power series, which are particular cases of series of functions and that have an important role for some practical applications; for example, using the power series we can find the approximate values of some functions so we can appreciate the precision of a computing method.

In the third chapter are treated some elements of the differentiation theory of functions.

The fourth chapter presents some elements of Vector Analysis with applications to physics and differential geometry.

The fifth chapter presents some notions of implicit functions and extremes of functions of one or more variables.

Chapter six is dedicated to integral calculus, which is useful to solve various geometric problems and to mathematical formulation of some concepts from physics.

Seventh chapter deals with the study of the differential equations and systems of differential equations that model the physical processes.

The chapter eight deals with the line and double integrals. The line integral is a generalization of the simple integral and allows the understanding of some concepts from physics and engineering; the double integral has a meaning analogous to that of the simple integral: like the simple definite integral is the area bordered by a curve, the double integral can be interpreted as the volume bounded by a surface.

The last chapter is dedicated to the triple and surface integral calculus. Although it is not possible a geometric interpretation of the triple integral, mechanically speaking, this integral can be interpreted as a mass, being considered as the distribution of the density in the respective space. The surface integral is a generalization of the double integral in some plane domains, as the line integral generalizes the simple definite integral.

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January 10, 2012

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# Contents

<b>1</b>	<b>Sequences and Series of Numbers</b> . . . . .	1
1.1	Cauchy Sequences . . . . .	1
1.2	Fundamental Concepts . . . . .	3
1.2.1	Convergent Series . . . . .	3
1.2.1.1	Cauchy's Test . . . . .	5
1.2.2	Divergent Series . . . . .	7
1.2.3	Operations on Convergent Series . . . . .	11
1.3	Tests for Convergence of Alternating Series . . . . .	14
1.4	Tests of Convergence and Divergence of Positive Series . . . . .	16
1.4.1	The Comparison Test I . . . . .	16
1.4.2	The Root Test . . . . .	19
1.4.3	The Ratio Test . . . . .	22
1.4.4	The Raabe's and Duhamel's Test . . . . .	26
1.4.5	The Comparison Test II . . . . .	29
1.4.6	The Comparison Test III . . . . .	30
1.5	Absolutely Convergent and Semi-convergent Series . . . . .	31
1.6	Problems . . . . .	34
<b>2</b>	<b>Power Series</b> . . . . .	41
2.1	Region of Convergence . . . . .	41
2.2	Taylor and Mac Laurin Series . . . . .	49
2.2.1	Expanding a Function in a Power Series . . . . .	49
2.3	Sum of a Power Series . . . . .	60
2.4	Problems . . . . .	65

<b>3</b>	<b>Differentiation Theory of the Functions</b> . . . . .	71
3.1	Partial Derivatives and Differentiable Functions of Several Variables . . . . .	71
3.1.1	Partial Derivatives . . . . .	71
3.1.2	The Total Differential of a Function . . . . .	83
3.1.3	Applying the Total Differential of a Function to Approximate Calculations . . . . .	90
3.1.4	The Functional Determinant . . . . .	93
3.1.5	Homogeneous Functions . . . . .	99
3.2	Derivation and Differentiation of Composite Functions of Several Variables . . . . .	102
3.3	Change of Variables . . . . .	119
3.4	Taylor's Formula for Functions of Two Variables . . . . .	126
3.5	Problems . . . . .	143
<b>4</b>	<b>Fundamentals of Field Theory</b> . . . . .	157
4.1	Derivative in a Given Direction of a Function . . . . .	157
4.2	Differential Operators . . . . .	162
4.3	Problems . . . . .	179
<b>5</b>	<b>Implicit Functions</b> . . . . .	187
5.1	Derivative of Implicit Functions . . . . .	187
5.2	Differentiation of Implicit Functions . . . . .	193
5.3	Systems of Implicit Functions . . . . .	203
5.4	Functional Dependence . . . . .	209
5.5	Extreme Value of a Function of Several Variables Conditional Extremum . . . . .	214
5.6	Problems . . . . .	229
<b>6</b>	<b>Terminology about Integral Calculus</b> . . . . .	245
6.1	Indefinite Integrals . . . . .	245
6.1.1	Integrals of Rational Functions . . . . .	245
6.1.2	Reducible Integrals to Integrals of Rational Functions . . . . .	251
6.1.2.1	Integrating Trigonometric Functions . . . . .	251
6.1.2.2	Integrating Certain Irrational Functions . . . . .	252
6.2	Some Applications of the Definite Integrals in Geometry and Physics . . . . .	260
6.2.1	The Area under a Curve . . . . .	260
6.2.2	The Area between by Two Curves . . . . .	265
6.2.3	Arc Length of a Curve . . . . .	269
6.2.4	Area of a Surface of Revolution . . . . .	274
6.2.5	Volumes of Solids . . . . .	276
6.2.6	Centre of Gravity . . . . .	277

6.3	Improper Integrals . . . . .	280
6.3.1	Integrals of Unbounded Functions . . . . .	280
6.3.2	Integrals with Infinite Limits . . . . .	288
6.3.3	The Comparison Criterion for the Integrals . . . . .	293
6.4	Parameter Integrals . . . . .	296
6.5	Problems . . . . .	302
<b>7</b>	<b>Equations and Systems of Linear Ordinary</b>	
	<b>Differential Equations . . . . .</b>	<b>317</b>
7.1	Successive Approximation Method . . . . .	317
7.2	First Order Differential Equations Solvable	
	by Quadratures . . . . .	320
7.2.1	First Order Differential Equations with Separable	
	Variables . . . . .	322
7.2.2	First Order Homogeneous Differential Equations . . .	324
7.2.3	Equations with Reduce to Homogeneous	
	Equations . . . . .	326
7.2.4	First Order Linear Differential Equations . . . . .	330
7.2.5	Exact Differential Equations . . . . .	332
7.2.6	Bernoulli's Equation . . . . .	339
7.2.7	Riccati's Equation . . . . .	340
7.2.8	Lagrange's Equation . . . . .	342
7.2.9	Clairaut's Equation . . . . .	346
7.3	Higher Order Differential Equations . . . . .	349
7.3.1	Homogeneous Linear Differential Equations	
	with Constant Coefficients . . . . .	349
7.3.2	Non-homogeneous Linear Differential Equations	
	with Constant Coefficients . . . . .	357
	7.3.2.1 The Method of Variation of Constants . . . . .	357
	7.3.2.2 The Method of the Undetermined	
	Coefficients . . . . .	360
7.3.3	Euler's Equation . . . . .	367
7.3.4	Homogeneous Systems of Differential Equations	
	with Constant Coefficients . . . . .	369
7.3.5	Method of Characteristic Equation . . . . .	371
7.3.6	Elimination Method . . . . .	373
7.4	Non-homogeneous Systems of Differential Equations	
	with Constant Coefficients . . . . .	377
7.5	Problems . . . . .	380
<b>8</b>	<b>Line and Double Integral Calculus . . . . .</b>	<b>395</b>
8.1	Line Integrals of the First Type . . . . .	395
8.1.1	Applications of Line Integral of the First Type . . . . .	396
8.2	Line Integrals of the Second Type . . . . .	406
8.3	Calculus Way of the Double Integrals . . . . .	421

8.4	Applications of the Double Integral . . . . .	430
8.4.1	Computing Areas . . . . .	430
8.4.2	Mass of a Plane Plate . . . . .	431
8.4.3	Coordinates the Centre of Gravity of a Plane Plate . . . . .	433
8.4.4	Moments of Inertia of a Plane Plate . . . . .	436
8.4.5	Computing Volumes . . . . .	438
8.5	Change of Variables in Double Integrals . . . . .	441
8.5.1	Change of Variables in Polar Coordinates . . . . .	441
8.5.2	Change of Variables in Generalized Polar Coordinates . . . . .	445
8.6	Riemann-Green Formula . . . . .	447
8.7	Problems . . . . .	454
<b>9</b>	<b>Triple and Surface Integral Calculus . . . . .</b>	<b>475</b>
9.1	Calculus Way of the Triple Integrals . . . . .	475
9.2	Change of Variables in Triple Integrals . . . . .	477
9.2.1	Change of Variables in Spherical Coordinates . . . . .	477
9.2.2	Change of Variables in Cylindrical Coordinates . . . . .	480
9.3	Applications of the Triple Integrals . . . . .	485
9.3.1	Mass of a Solid . . . . .	485
9.3.2	Volume of a Solid . . . . .	493
9.3.3	Centre of Gravity . . . . .	499
9.3.4	Moments of Inertia . . . . .	509
9.4	Surface Integral of the First Type . . . . .	512
9.5	Surface Integral of the Second Type . . . . .	520
9.5.1	Flux of a Vector Field . . . . .	524
9.5.2	Gauss- Ostrogradski Formula . . . . .	529
9.5.3	Stokes Formula . . . . .	540
9.6	Problems . . . . .	551
	<b>References . . . . .</b>	<b>573</b>
	<b>List of Symbols . . . . .</b>	<b>577</b>
	<b>Index . . . . .</b>	<b>579</b>