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Analytic Methods of Sound Field Synthesis

 Springer

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Preface

The present book summarizes the work that I have performed in the context of my doctoral dissertation and my subsequent activities at the Quality and Usability Lab, which is jointly run by the University of Technology Berlin and Deutsche Telekom Laboratories. The initial motivation for this work has been the question of how the two best-known methods of sound field synthesis, namely Wave Field Synthesis and Near-field Compensated Higher Order Ambisonics, relate. The answer to this question had been discussed in the research communities for years but a convincing conclusion had not been found. I present in this book a general formulation for the problem of sound field synthesis that allows for identifying above methods as particular solutions so that a juxtaposition is straightforward. Practical applications and synthesis of sound fields with diverse properties are then treated based on the general framework, which further facilitates the interpretation. The website <http://www.soundfieldsynthesis.org> accompanying this book makes available for download MATLAB/Octave scripts for all included simulations so that the reader can perform further investigations without having to start from scratch.

As with any book, the people who deserve acknowledgements are too numerous to list. I therefore mention only those who receive my very special acknowledgements. All others who have contributed to my research work and who are not mentioned here shall be aware of my appreciation.

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Berlin, August 2011

Jens Ahrens

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Symbols

c	speed of sound in air ($c = 343\text{m/s}$ is assumed)
i	imaginary unit, $i^2 = -1$
$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	position vector in Cartesian coordinates (Appendix A)
$\mathbf{x}^T = [x \ y \ z]$	transposition of vector \mathbf{x}
$ x $	absolute value (Weisstein 2002)
$\ \mathbf{x}\ $	vector norm (Weisstein 2002)
\mathbf{e}_x	unit vector pointing in x -direction
(α, β)	direction specified by azimuth α and colatitude β
\arccos	inverse cosine (Weisstein 2002)
$\delta(\cdot)$	Dirac delta function
δ_{mn}	Kronecker Delta, defined in (2.26)
$\partial\Omega$	boundary enclosing volume Ω_i
Ω_i	volume enclosed by boundary $\partial\Omega$
Ω_e	domain exterior to boundary $\partial\Omega$
∇	gradient defined in (2.5) and (2.12)
$\Re\{\cdot\}$	real part
$\Im\{\cdot\}$	imaginary part
$G(\mathbf{x}, \mathbf{x}_0, \omega)$	Green's function for excitation at \mathbf{x}_0
$G_0(\mathbf{x} - \mathbf{x}_0, \omega)$	free-field Green's function for excitation at \mathbf{x}_0 given by (2.66)
$j_n(\cdot)$	n -th order spherical Bessel function (Arfken and Weber 2005)
$y_n(\cdot)$	n -th order spherical Neumann function (Arfken and Weber 2005)
$h_n^{(1,2)}(\cdot)$	n -th order spherical Hankel function of first and second kind defined in (2.15)
$Y_n^m(\beta, \alpha)$	spherical harmonic of n -th degree and m -th order, defined in (2.23)

$\check{S}_n^m(r, \omega)$	spherical harmonics expansion coefficients of sound field $S(\mathbf{x}, \omega)$, defined in (2.31)
$\check{S}_{n,i}^m(\omega)$ or $\check{S}_n^m(\omega)$	interior expansion coefficients of sound field $S(\mathbf{x}, \omega)$, defined in (2.32a)
$\check{S}_{n,e}^m(\omega)$	exterior expansion coefficients of sound field $S(\mathbf{x}, \omega)$, defined in (2.32b)
$\check{S}(k_x, k_y, z, \omega)$	sound field $S(\mathbf{x}, \omega)$ considered in wavenumber domain with respect to k_x and k_y
$\check{S}(\cdot)$	angular spectrum representation of $S(\cdot)$, defined in (2.55a)
$\bar{S}(\cdot)$	signature function of sound field $S(\cdot)$, defined in (2.45)
$P_n^m(\cdot)$	associated Legendre function of n -th degree and m -th order (Gumerov and Duraiswami 2004)
S_R^2	2-sphere (i.e. a spherical surface) of radius R (Weisstein 2002)
S_R^1	1-sphere (i.e. a circle) of radius R (Weisstein 2002)
$(I I)_{nn'}^{mm'}(\cdot)$	translation coefficient for interior-to-interior translation
$(E I)_{nn'}^{mm'}(\cdot)$	translation coefficient for exterior-to-interior translation
$\text{sinc } x$	sinus cardinalis, $\text{sinc } x = \sin(\pi x)/(\pi x)$
$\langle \cdot \rangle$	inner product (Weisstein 2002)
$\gamma_{n_1, n_2, n}^{m_1, m_2, m}$	Gaunt coefficient, defined in (D.6)
$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$	Wigner 3j-Symbol as defined in (Weisstein 2002)
$\mathcal{E} \begin{pmatrix} m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{pmatrix}$	E-symbol, defined in (D.7)
${}_3F_2(\cdot)$	generalized hypergeometric function (Arfken and Weber 2005)
$(\cdot)!$	factorial (Weisstein 2002)
$(\cdot)!!$	double factorial (Weisstein 2002)
$\frac{\partial}{\partial \mathbf{n}}$	gradient in direction \mathbf{n} , refer to (2.61)
$dA(\mathbf{x}_0)$	infinitesimal surface element
$S(\omega)^*$	complex conjugate of $S(\omega)$
$\circ \text{---} \bullet$	Fourier transform
$\bullet \text{---} \circ$	inverse Fourier transform
$[x]$	floor function, gives the largest integer not greater than x (Weisstein 2002)
$\lceil x \rceil$	ceiling function, gives the smallest integer not smaller than x (Weisstein 2002)
$\text{sign}(x)$	signum function, defined in (5.73)
$J_m(\cdot)$	Bessel function of m -th order (Weisstein 2002)
$H_m^{(2)}(\cdot)$	m -th order Hankel function of second kind (Weisstein 2002)

Acronyms

ASW	Apparent Source Width
ASDF	Audio Scene Description Format
BRIR	Binaural Room Impulse Response
BRTF	Binaural Room Transfer Function
FIR	Finite Impulse Response
FOS	First-Order Section
HOA	Higher Order Ambisonics
HRIR	Head-related Impulse Response
HRTF	Head-related Transfer Function
IIR	Infinite Impulse Response
LEV	Listener Envelopment
NFC-HOA	Near-field Compensated Higher Order Ambisonics
SDM	Spectral Division Method
SOS	Second-Order Section
SpatDIF	Spatial Sound Description Interchange Format
WFS	Wave Field Synthesis