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# Automated Deduction in Geometry

8th International Workshop, ADG 2010  
Munich, Germany, July 22-24, 2010  
Revised Selected Papers

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# Preface

From July 22 to July 24, 2010, the Technische Universität München, Germany, hosted the eighth edition of the now well-established ADG workshop dedicated to Automatic Deduction in Geometry. From the first edition, which was held in Toulouse in 1996, to ADG 2010, a slow mutation has taken place. The workshop that was formerly centered around computer algebra became a larger forum where several communities could exchange new ideas coming from various domains, such as computer algebra, logic, computer-assisted proof, combinatorial geometry or even software development, but all focused on proof in geometry.

ADG 2010 was a fruitful meeting where 19 papers, from 22 submissions, were selected for presentation after a review process involving at least two reviewers per article. The set of presentations was completed by an invited talk given by Robert Joan-Arinyo from the Universitat Politècnica de Catalunya, Spain. ADG 2010 was also an enjoyable meeting thanks to the rigorous and flawless organization of the Munich team (see the Organizing Committee list).

After the meeting, a new call for papers was launched, accepting contributions not necessarily related to a presentation at ADG 2010.

The present volume of the LNAI series is the result of this selection process, which includes a new review process and discussions within the Program Committee. It is composed of 13 papers which present original research reflecting the current state of the art in this field. The following categorization proposes a key to understanding the papers. But, obviously as with all categorizations, it is rather arbitrary and it should not be taken strictly. Most papers can indeed also be considered from a radically different point of view.

Three papers deal with incidence geometry using some kind of combinatoric argument. Susanne Apel and Jürgen Richter-Gerbert explore two ways to automatically prove a geometric theorem by discovering cancellation patterns. Dominique Michelucci studies incidence geometry leading to two papers: one deals with an abstract notion of line and the other concerns human readable proofs in geometry.

Three papers fall in the domain of computer algebra. Daniel Lichtblau studies a problem related to the locus of the midpoint of a triangle in a corner, which is a variant of the “penny in a corner” problem, by using numeric, formal and graphical tools. Pavel Pech exposes a method to automatically prove theorems related to inequalities in geometry. Yu Zou and Jingzhong Zhang propose a way to generate readable proofs using the so-called Mass Point Method involving barycentric calculations with real or complex masses.

Four papers are more related to software implementation. Michael Gerhäuser and Alfred Wassermann present a Web-integrated software for dynamic geometry which includes a Gröbner-based tool able to compute plane loci. Fadoua Ghourabi, Tetsuo Ida and Asem Kasem expose methods to produce readable

proofs of theorem within the Origami problematics. Pedro Quaresma describes TGTP—a library of problems for automated theorem proving in geometry. Phil Scott and Jacques Fleuriot present the concurrent implementation of a forward chaining algorithm in the Isabelle/HOL framework.

Last but not least, logic and proof assistants are the subject of three papers of this book. Following his own work on non-standard analysis, Jacques Fleuriot explores the foundations of discrete geometry in Isabelle/HOL. Laurent Fuchs and Laurent Théry represent here both the Coq and the geometric algebra communities by presenting the formalization in Coq of Grassmann Caley Algebra and its application to automatize the production of proofs in projective geometry. Sana Stojanović, Vesna Pavlović and Predrag Janičić expose a framework where coherent logic is used to implement a geometric prover able to deliver readable proofs.

Our gratitude goes to the Chairs of the previous editions of ADG. We thank them for their guidance and for having made ADG what it is now. We would also to thank the Program Committee and the numerous referees who did a lot of work to improve the quality of the workshop and of this book.

July 2011

Pascal Schreck

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