

# **NONLINEAR PHYSICAL SCIENCE**

# NONLINEAR PHYSICAL SCIENCE

*Nonlinear Physical Science* focuses on recent advances of fundamental theories and principles, analytical and symbolic approaches, as well as computational techniques in nonlinear physical science and nonlinear mathematics with engineering applications.

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# **Waves and Structures in Nonlinear Nondispersive Media**

General Theory and Applications to Nonlinear  
Acoustics

With 181 figures



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ISSN 1867-8440

e-ISSN 1867-8459

Nonlinear Physical Science

ISBN 978-7-04-031695-7

Higher Education Press, Beijing

ISBN 978-3-642-23616-7

e-ISBN 978-3-642-23617-4

Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2011935427

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# Preface

The book is aimed at natural science undergraduates, as well as at graduate and post-graduate students studying the theory of nonlinear waves of various physical nature. It may also be useful as a handbook for engineers and researchers who encounter the necessity of taking nonlinear wave effects into account in their work.

Evolution of sufficiently intense waves is determined by nonlinear processes, in which the progress is substantially influenced by dispersion (a dependence of the phase velocity on its frequency). Media without dispersion, where the phase velocity does not depend on the frequency, are the simplest ones with respect to their physical properties and are the most common in nature. But nonlinear interactions of the Fourier spectral components in such media are particularly complex and diverse. Here, practically all “virtual” energy-exchange processes between waves of different frequencies become resonant ones and occur with a high efficiency. An avalanche-like increase of the number of spectral components of the field takes place, which, within the space-time representation, corresponds to formation of structures with strongly pronounced nonlinear properties. Examples of such structures are discontinuities of a function describing the wave field or discontinuities of its derivative, steep shock fronts of various types and multidimensional cellular structures.

Nonlinear structures can be stable only in strong fields, under the conditions of competition with effects of absorption, dispersion, etc, which contribute to the decay of such structures. These objects have properties of quasiparticles. For instance, shock fronts undergo inelastic collisions. Thus, in nondispersive media, nonlinearity provides both a possibility of interactions between stable structures and their very existence. Solitons are other well-known objects in nonlinear physics, which are, generally speaking, stable only in idealized conservative systems. At the same time, quasi stability of shock-front structures or sawtooth waves occurs in real dissipative systems.

Structures of different physical nature are described by similar mathematical models. These models are used not only in the wave theory, but also to describe various non-wave objects, *viz.*: forest-fire fronts, density of a flow of non-interacting particles, etc. Because of the universality of such nonlinear models, it is necessary to

analyze them on the basis of general principles of mathematical physics, irrespective of the nature of the described phenomena.

On the other hand, nondispersive waves and structures are widely used in science and technology. A review of these applications, from the authors' viewpoint, is what "brightens up" the theory and may be of interest to many readers.

The theory of nonlinear waves and structures is a very extensive and constant developing field of physics (especially radiophysics and mathematical physics). It has many specific applications. Among them there are both the well-known problems of acoustics, electrodynamics and plasma physics (see, e.g., [1–5]), and the less-known problems, such as surface-growth description [6, 7], dynamics of turbulence [8, 9] and development of a gravitational instability of the large-scale distribution of matter in the Universe [10–14]. A wide range of phenomena arising here have led to the development of a variety of mathematical methods, which are effective in addressing various kinds of nonlinear fields and waves (see, e.g., [15–17]). It is clear that within a single monograph, it is not possible to give an exhaustively comprehensive overview of the whole problem. For this reason, the authors limited themselves to a discussion of the "hydrodynamic" type of nonlinear waves in nondispersive medium. First of all, the properties of solutions to such standard nonlinear wave equations in nondispersive media as the simple wave equation, the Burgers equation and the Kardar-Parisi-Zhang equation have been studied in detail. Apart from the importance of these equations for the theory and applications, an analysis of these solutions allows us to trace stages of development of typical nonlinear processes and, above all, nonlinear distortion of profiles, the gradient catastrophe and emergence of shock waves. In order for the theory of nonlinear waves in nondispersive media not to look too abstract, the presentation is based on illustrative geometric interpretations of both the equations themselves and their solutions, as well as on a comprehensive discussion of the physical meaning of these solutions and the methods used to obtain them.

The monograph consists of two parts. The first part is devoted to a detailed description of the concepts and analysis methods of nonlinear waves and structures in nondispersive media. The second part focuses on an in-depth description of the nonlinear theory as applied only to one type of waves — high intensity acoustic waves. This object, on the one hand, is the most straightforward and, on the other hand, has important practical applications.

The authors have attempted to communicate all materials at the following "two levels" of complexity. The first level is intended to introduce beginning investigators (above all undergraduate, graduate and PhD students) to the concepts and methods of the theory of nonlinear waves and structures in nondispersive media. In order to achieve a deeper understanding of the foundations, it is useful to solve the problems given in the end of the chapters in Part I. The second, higher, level is meant for researchers, who already have experience in this field of study and are interested in the state of the art or in specific results. Naturally, it is impossible to reflect the entire diversity of approaches used to study nonlinear fields and waves in a single monograph. This is why the material is presented at a simple, "physical" level of rigor, where possible. Those, who are interested in a more rigor-

ous mathematical foundation of the problems discussed here, are advised to turn to monographs [15, 17], where mathematical foundations of many topics touched upon in this book are thoroughly discussed. An in-depth review of the methods used to solve nonlinear problems, along with profound results of the nonlinear field theory, can be found in book [16]. In monograph [18], and also in textbook [19], the theory of generalized functions necessary for construction of generalized solutions of nonlinear equations is comprehensively elucidated. We recommend those who intend deeper to delve into the nonlinear field theory, without burying themselves in mathematical subtleties, the following thorough monographs and textbooks: [1, 2, 4, 5], which are written by physicists for physicists. Basic concepts of the nonlinear wave theory, along with illustrative physical examples, can be found in the remarkable textbook [14]. To those who are going professionally to engage themselves in the field of nonlinear acoustics, we recommend monograph [3] and the books of problems [20, 21], where a set of problems aiding in mastering various aspects of nonlinear acoustics is given. If one is interested in statistical properties of nonlinear random waves as applied to nonlinear acoustics, astrophysics and turbulence, he or she can pick up necessary information from monograph [10]. We also advise to turn to monograph [8], which covers the foundations of the theory of strong turbulence and its inherent phenomena, such as intermittency and multifractality.

We are grateful to the renowned scientists, fruitful interactions with whom over the years have formed our vision of the problems and methods of the nonlinear science. First of all, they are: academicians A.V. Gaponov-Grekhov, Ya.B. Zeldovich, R.V. Khokhlov, V.I. Arnold and Ya.G. Sinai; corresponding members of the Russian Academy of Sciences M.I. Rabinovich and D.I. Trubetskov; Professors A.N. Malakhov, L.A. Ostrovsky, S.A. Rybak, S.I. Soluyan, A.P. Sukhorukov, A.S. Chirkin and S.F. Shandarin. We are delighted to remember the years of collaboration with international colleagues, among whom are: D. Crighton, U. Frisch, B. Enflo, D. Blackstock, M. Hamilton, L. Cram, E. Aurell, A. Noullez, W.A. Woyczynski and many others.

We would like also to thank our translators, O. Simdyankina and S. Simdyankin, not only for the speedy production of an English translation of this book, but also for the lucid clarity of their literary representation of the original text.

Nizhny Novgorod, Moscow,  
July 2011

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## References

1. M.J. Lighthill, *Waves in Fluids*, 2nd edn. (Cambridge University Press, 2002)
2. M.I. Rabinovich, D.I. Trubetskov, *Oscillations and Waves in Linear and Nonlinear Systems*. (Springer, Berlin, 1989)
3. O.V. Rudenko, S.I. Soluyan, *Theoretical Foundations of Nonlinear Acoustics* (Plenum, New York, 1977)

4. M.B. Vinogradova, O.V. Rudenko, A.P. Sukhorukov, *Theory of Waves* (Nauka, Moscow, 1979). In Russian
5. G.B. Whitham, *Linear and Nonlinear Waves* (Wiley, New York, 1974)
6. A.L. Barabási, H.E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, 1995)
7. M. Kardar, G. Parisi, Y.C. Zhang, Dynamical scaling of growing interfaces, *Phys. Rev. Lett.* **56**, 889–892 (1986)
8. U. Frisch, *Turbulence: The Legacy of A.N. Kolmogorov* (Cambridge University Press, 1995)
9. A.S. Monin, A.M. Yaglom, *Statistical Fluid Mechanics*, vol. 2 (MIT Press, Cambridge, Mass, 1975)
10. S.N. Gurbatov, A.N. Malakhov, A.I. Saichev, *Nonlinear Random Waves and Turbulence in Nondispersive Media: Waves, Rays and Particles*. (Manchester University Press, 1991)
11. P.J.E. Peebles, *Large-Scale Structure of the Universe* (Princeton University Press, 1980)
12. S.F. Shandarin, Y.B. Zeldovich, The large-scale structure of the universe: turbulence, intermittency, structures in a self-gravitating medium, *Rev. Mod. Phys.* **61**, 185–220 (1989)
13. S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972)
14. Y.B. Zeldovich, *Elements of Mathematical Physics* (Nauka, Moscow, 1973). In Russian
15. V.I. Arnold, *Ordinary Differential Equations* (MIT Press, Cambridge, Mass, 1978)
16. R. Richtmyer, *Principles of Advanced Mathematical Physics*, vol. 1 (Springer, Berlin, 1978)
17. B.L. Rozhddestvenskii, N.N. Yanenko, *Systems of Quasilinear Equations* (Nauka, Moscow, 1978). In Russian
18. A.I. Saichev, W.A. Woyczyński, *Distributions in the Physical and Engineering Sciences* (Birkhäuser, Boston, 1997)
19. S.A. Lapinova, A.I. Saichev, V.A. Filimonov, *Generalized Functions and Asymptotic Methods* (Nizhny Novgorod University Press, 2006). In Russian
20. S.N. Gurbatov, O.V. Rudenko (eds.), *Acoustics in Problems* (Fizmatlit, Moscow, 2009). In Russian
21. O.V. Rudenko, S.N. Gurbatov, C.M. Hedberg, *Nonlinear Acoustics through Problems and Examples* (Trafford, 2010)



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