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Modelling Water Flow in Unsaturated Porous Media

Accounting for Nonlinear Permeability and Material Heterogeneity

Springer

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Contents

Notations

Subscripts related to fluid phases:

a	Air,
w	Water,
α	Generic fluid phase.

Subscripts related to the components of vectors and tensors (Einstein summation convention is implied only for these subscripts): i, j, k, l, m, n.

Superscripts related to spatial discretization (in parentheses)

(i) to (m)	Indices of grid entities: nodes, elements, volumes or faces,
(ij)	Average value between nodes i and j,
(e)	Average value in a finite element,
(fc)	Average value at a finite volume face,
(τ)	Time step index,
(v)	Iteration index.

Superscripts related to components of a heterogeneous medium:

I	Fracture system or background material
II	Rock matrix or inclusions

Superscripts related to averaged field-scale properties:

eff	Effective parameter,
eq	Equivalent parameter.

Superscripts related to homogenization analysis:

$*$	Dimensionless variable,
(c)	Characteristic value.

Symbols (physical units specified where appropriate):

Uppercase Latin letters:

$A_{\alpha}^{(ij)}$	Coefficients in spatially discretized flow equations for phase α at node j,
B_{ij}^{high}	Geometry-dependent constant for calculating the effective water permeability in a medium with highly permeable inclusions,
B_{ij}^{low}	Geometry-dependent constant for calculating the effective water permeability in a medium with weakly permeable inclusions,
$C_{\text{it}}^{\text{dec}}$	Coefficient decreasing the time step size,
$C_{\text{it}}^{\text{inc}}$	Coefficient increasing the time step size,
C_{ch}	Specific water capacity in the pressure head based form of the Richards equation, (m^{-1}),
C_{wh}	Storage coefficient in the pressure head based form of the Richards equation, (m^{-1}),
C_{wp}	Storage coefficient in the pressure based form of the Richards equation, (Pa^{-1}),
D	Hydraulic diffusivity tensor, ($\text{m}^2\,\text{s}^{-1}$),
D_{e}	Energy dissipation for fluid flow in porous medium, (Pa^2),
D_L	Characteristic diffusivity at the field scale (m),
D_l	Characteristic diffusivity at the Darcy scale (m),
$E^{(n)}$	nth finite element,
$F^{(n)}$	nth face of a finite volume grid,
$G_{\alpha}^{(j)}$	Gravity term in spatially discretized flow equation for phase α at node j,
H_{α}	Total potential head of fluid phase α, (m),
$K_{\text{s}\alpha}$	Hydraulic conductivity of phase α at apparent saturation, ($\text{m}\,\text{s}^{-1}$),
K_{α}	Hydraulic conductivity of phase α, ($\text{m}\,\text{s}^{-1}$),
L	Characteristic length at the field scale, (m),
M_{α}	Mass density of fluid phase α with respect to the bulk volume of porous medium, $M_{\alpha} = \rho_{\alpha}\,\phi_{\alpha}\,S_{\alpha,}$, ($\text{kg}\,\text{m}^{-3}$),
$Q_{\alpha}^{(\text{fc})}$	Total mass flux of phase α at a control volume face, ($\text{kg}\,\text{s}^{-1}$),
R_{b}	Characteristic dimension of a matrix block or inclusion, (m),
R_{gas}	Universal gas constant, $(\text{J}(\text{mol}\,^{\circ}\text{K})^{-1})$,
$S_{\text{a}}^{\text{trap}}$	Effective air saturation for a heterogeneous medium in trapped-air regime,
$S_{\text{w}}^{\text{trap}}$	Effective water saturation for a heterogeneous medium in trapped-air regime,
S_{α}	Saturation of fluid phase α,
$S_{\text{e}\alpha}$	Normalized saturation of phase α,
$S_{\alpha}^{\max}$	Maximum attainable saturation of phase α,
$S_{\alpha}^{\min}$	Minimum attainable saturation of phase α,

$S_{r\alpha}$	Saturation of phase α at residual state,
T_{α}^{m}	Non-equilibrium mass flow rate between fractures and matrix, or between background material and inclusions, $(\mathrm{kg}(\mathrm{m}^3\,\mathrm{s})^{-1})$,
T_{α}^{v}	Non-equilibrium volumetric flow rate between fractures and matrix, or between background material and inclusions, (s^{-1}),
U	Domain of a representative elementary volume at the pore scale,
U_{α}	Part of a pore-scale representative elementary volume occupied by phase α,
V_{b}	Matrix block domain,
$V^{(i)}$	ith finite volume,
W	Solution domain at the Darcy scale,
Z_{k}	Auxiliary parameter in the formula for inter-nodal permeability,
Z_{s}	Auxiliary parameter in the formula for relating parameters of the Brook-Corey and van Genuchten functions.

Calligraphic Latin letters:

$\mathscr{D}^{(j)}$	Spatial discretization operator for node j,
$\mathscr{E}^{(j)}$	Set of finite elements sharing node j,
$\mathscr{H}$	Relative air humidity,
$\mathscr{M}_{\alpha}$	Mole mass of fluid phase α, $(\mathrm{kg\ mol}^{-1})$,
$\mathscr{N}_{\mathrm{elem}}^{(n)}$	Set of nodes belonging to n-th finite element,
$\mathscr{N}_{\mathrm{nod}}^{(j)}$	Set of nodes connected to node j, including j,
$\mathscr{O}(u)$	Order of magnitude of number u,
$\mathscr{R}_{c}$	Dimensionless gravity–capillarity ratio,
$\mathscr{R}_{d}$	Dimensionless hydraulic diffusivity ratio between inclusions and background,
$\mathscr{R}_{k}$	Dimensionless permeability ratio between inclusions and background,
$\mathscr{R}_{t}^{\iota}$	Dimensionless time scale for Darcy-scale flow in porous material ι,
$\mathscr{T}$	Kelvin temperature, (°K).

Lowercase Latin letters:

a_i	Length of the ellipsoidal inclusion in ith spatial direction, (m),
a', a'_{10}, a'_{11}, a'_{2}	Parameters in the Gasto et al. formula for the inter-nodal permeability,
b', b'_{01}, b'_{02}, b'_{1}	Parameters in the Gasto et al. formula for the inter-nodal permeability,
c', c'_{0}	Parameters in the Gasto et al. formula for the inter-nodal permeability,
d_i	Ellipsoid depolarisation coefficient in ith spatial direction,
f_{α}	Fractional flow function for fluid phase α,
g	Magnitude of the gravitational acceleration vector, $(\mathrm{m\ s}^{-2})$,

$\mathbf{g}$	Gravitational acceleration vector, (m s^{-2}),
h_α	Pressure head of fluid phase α, (Pa),
h_c	Capillary pressure head, (Pa),
h_e	Air-entry pressure head, (Pa),
h_g	Pressure head scaling parameter, (Pa),
$k_{ca}^{(ij)}$	Average relative water permeability for the capillary-driven flow between nodes i and j,
$k_{gr}^{(ij)}$	Average relative water permeability for the gravity-driven flow between nodes i and j,
$k_{int}^{(ij)}$	Integrated average relative water permeability between nodes i and j,
k_{ii}^{cpl}	Cardwell and Parsons lower bound for the equivalent permeability in ith direction, $i = 1, 2, 3$, (m^2),
k_{ii}^{cpu}	Cardwell and Parsons upper bound for the equivalent permeability in ith direction, $i = 1, 2, 3$, (m^2),
$k_{rw}^{(el)}$	Average relative water permeability in a finite element,
k_{rw}^{fm}	Relative water permeability at the fracture–matrix interface,
$k_{rw}^{(ij)}$	Average relative water permeability between nodes i and j,
$k_{r\alpha}$	Relative permeability of phase α,
k_s	Intrinsic permeability tensor, (m^2),
k_s^{eq}	Equivalent intrinsic permeability tensor of heterogeneous medium, (m^2),
k_s^{eff}	Effective intrinsic permeability tensor of a heterogeneous medium, (m^2),
k_t	Total permeability tensor in fractional flow formulation, ($m^2(Pa\,s)^{-1}$),
k_α	Permeability tensor of phase α, (m^2),
k_α^{eff}	Effective permeability tensor of a heterogeneous medium for phase α, (m^2),
k_w^{high}	Effective water permeability tensor for a heterogeneous medium with highly permeable inclusions, (m^2),
k_w^{low}	Effective water permeability tensor for a heterogeneous medium with weakly permeable inclusions, (m^2),
k_w^{trap}	Effective water permeability tensor for a heterogeneous medium in trapped-air regime, (m^2),
l	Characteristic length at the Darcy scale, (m),
l_h	Characteristic dimension of Darcy-scale heterogeneities, (m),
l_v	Characteristic dimension of the averaging volume, (m),
m_g	Exponent in the van Genuchten capillary function,
n_b	Exponent in the Brooks–Corey capillary function,
n_g	Exponent in the van Genuchten capillary function,

$\boldsymbol{n}_E$	Unit vector normal to the boundary of a finite element,
$\boldsymbol{n}_V$	Unit vector normal to the boundary of a finite volume,
$\boldsymbol{n}_W$	Unit vector normal to the boundary of the porous domain,
$\boldsymbol{n}$	Unit vector normal to the interface separating two porous materials,
p_{atm}	Atmospheric pressure, (Pa),
p_{c}	Capillary pressure, (Pa),
p_{e}	Air-entry pressure, (Pa),
$p_{\mathrm{e}}^{\mathrm{drain}}$	Air-entry pressure during drainage, (Pa),
$p_{\mathrm{e}}^{\mathrm{wet}}$	Air-entry pressure during wetting, (Pa),
p_{g}	Capillary pressure scaling parameter in the Gardner and van Genuchten functions, (Pa),
p_α	Pressure in fluid phase α, (Pa),
p_α^{ref}	Reference pressure for fluid phase α, (Pa),
p_{glob}	Global pressure in the fractional flow formulation, (Pa),
$\tilde{p}_i$	Fluctuation of the fluid pressure for steady flow in ith spatial direction, (Pa),
q_{ev}	Cumulative evaporation flux at the soil surface, (m),
q_{inf}	Cumulative infiltration flux at the soil surface, (m),
r_{b}	Local spatial coordinate in a matrix block or inclusion, (m),
r_{c}	Radius of a capillary tube, (m),
$r_{\mathrm{c}1}, r_{\mathrm{c}2}$	Main curvature radii of the air–water interface, (m),
s_{abs}	Absolute error tolerance in the solution of nonlinear algebraic equations,
s_{rel}	Relative error tolerance in the solution of nonlinear algebraic equations,
t	Time, (s),
t_{dry}	Surface drying time in the evaporation simulation, (s),
t_{pond}	Surface ponding time in the infiltration simulation, (s),
u	Generic variable,
$\mathbf{u}$	Vector of unknown nodal values in the numerical solution,
v_L	Characteristic advective velocity at the field scale, $(\mathrm{m\,s^{-1}})$,
v_l	Characteristic advective velocity at the Darcy scale, $(\mathrm{m\,s^{-1}})$,
$v_{\mathrm{st}}^{(ij)}$	Steady-state volumetric water flux between nodes i and j, $(\mathrm{m\,s^{-1}})$,
$v_{\mathrm{w}}^{\mathrm{top}}$	Volumetric water flux at the soil surface, $(\mathrm{m\,s^{-1}})$,
$\mathbf{v}_\alpha$	Volumetric flux of fluid phase α with respect to the solid phase (Darcy velocity), $(\mathrm{m\,s^{-1}})$,
$\mathbf{v}_{\mathrm{t}}$	Total volumetric flux in the fractional flow formulation, $(\mathrm{m\,s^{-1}})$,
w	Volumetric fraction of a porous material,
$\mathbf{x}$	Spatial coordinate vector, (m),
$\mathbf{y}$	Spatial coordinate vector associated with a periodic cell, (m),
z	Elevation above the reference level, (m).

Uppercase Greek letters:

Γ	Interface between two porous materials,
$\Delta x^{(ij)}$	Distance between nodes i and j, (m),
$\Delta x'$	Normalized distance between nodes,
Δt	Time step, (s),
Θ	Weighting coefficient in the time discretization scheme,
Λ^{b}	External surface of a matrix block,
$\Lambda^{(i+1/2)}$	Interface between nested matrix blocks i and $i+1$ in MINC method,
Υ	Weighting function in the finite element method,
Φ_{h}	Flux potential with respect to the water pressure head, (m),
Φ_{p}	Flux potential with respect to the water pressure, (Pa),
$\Psi^{(i)}$	Shape function for node i in the finite element method,
$\Psi_{\mathrm{e}}^{(i)}$	Element shape function for node i in the finite element method,
Ω	Domain of a periodic cell,
Ω^{I}	Part of a periodic cell occupied by the background material,
Ω^{II}	Part of a periodic cell occupied by the inclusions.

Lowercase Greek letters:

α_{g}	Inverse of the scaling pressure (or pressure head) in the capillary function, (Pa^{-1}) or (m^{-1}),
$\alpha_{p,q}$	Coefficients in the modified equation,
β_{α}	Relative compressibility coefficient for fluid phase α, (Pa^{-1}),
β^{fm}	Shape coefficient for the fracture–matrix transfer term,
β'	Parameter in the averaging formula for the inter-nodal permeability,
$\beta_{p,q}$	Coefficients in the modified equation,
γ^{fm}	Scaling coefficient for the fracture–matrix transfer term,
$\gamma_{p,q}$	Coefficients in the modified equation,
$\delta\mathbf{u}$	Increment of the vector of unknown values in the iterative solution procedure,
ε	Scale parameter,
ζ	Gravity coefficient, cosine of the angle between x axis and the gravity vector in one-dimensional problems,
ζ'	Modified gravity coefficient,
η_{α}	Exponent in the power-law relative permeability function for phase α,
$\eta_1, \eta_2, \eta_3, \eta_4$	Exponents in the Mualem and Burdine relative permeability functions,
θ_{α}	Volumetric content of phase α,

$\theta_{r\alpha}$ Volumetric content of phase at the residual state α,
$\theta_{s\alpha}$ Volumetric content of phase α at the state of apparent saturation,
θ_a^{trap} Effective volumetric air content for a heterogeneous medium in trapped-air regime,
θ_w^{trap} Effective volumetric water content for a heterogeneous medium in trapped-air regime,
κ Connectivity parameter in the Mualem and Burdine relative permeability functions,
λ_α Mobility of phase α, $(\text{Pa s})^{-1}$,
μ_α Dynamic viscosity coefficient of fluid phase α, (Pa s),
$\boldsymbol{\xi}$ Local spatial coordinate in the finite element scheme,
π_i Weighting coefficient in the generalized power average formula for the equivalent permeability in ith spatial direction,
ρ_α Intrinsic mass density of fluid phase α, (kg m^{-3}),
ρ_α^{ref} Reference intrinsic mass density of fluid phase α, (kg m^{-3}),
$\sigma_{\alpha\beta}$ Surface tension between phases α and β, (N m^{-1}),
υ Small number used in numerical differentiation,
ϕ Porosity,
χ_i Auxiliary variable used to define the effective permeability in ith spatial direction,
Wetting angle,
ω_k Weighting parameter in the averaging scheme for fracture–matrix permeability,
ω_v Weighting parameter in the averaging formula for the inter-nodal relative permeability,
ω_w Weighting parameter in the averaging formula for the inter-nodal relative permeability.