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Adam Szymkiewicz

Modelling Water Flow in Unsaturated Porous Media

Accounting for Nonlinear Permeability
and Material Heterogeneity

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Notations

Subscripts related to fluid phases:

- a Air,
- w Water,
- α Generic fluid phase.

Subscripts related to the components of vectors and tensors (Einstein summation convention is implied only for these subscripts): i, j, k, l, m, n .

Superscripts related to spatial discretization (in parentheses)

- (i) to (m) Indices of grid entities: nodes, elements, volumes or faces,
- (ij) Average value between nodes i and j ,
- (e) Average value in a finite element,
- (fc) Average value at a finite volume face,
- (τ) Time step index,
- (v) Iteration index.

Superscripts related to components of a heterogeneous medium:

- I Fracture system or background material
- II Rock matrix or inclusions

Superscripts related to averaged field-scale properties:

- eff Effective parameter,
- eq Equivalent parameter.

Superscripts related to homogenization analysis:

- * Dimensionless variable,
- (c) Characteristic value.

Symbols (physical units specified where appropriate):

Uppercase Latin letters:

$A_{\alpha}^{(ij)}$	Coefficients in spatially discretized flow equations for phase α at node j ,
B_{ij}^{high}	Geometry-dependent constant for calculating the effective water permeability in a medium with highly permeable inclusions,
B_{ij}^{low}	Geometry-dependent constant for calculating the effective water permeability in a medium with weakly permeable inclusions,
$C_{\text{it}}^{\text{dec}}$	Coefficient decreasing the time step size,
$C_{\text{it}}^{\text{inc}}$	Coefficient increasing the time step size,
C_{ch}	Specific water capacity in the pressure head based form of the Richards equation, (m^{-1}),
C_{wh}	Storage coefficient in the pressure head based form of the Richards equation, (m^{-1}),
C_{wp}	Storage coefficient in the pressure based form of the Richards equation, (Pa^{-1}),
D	Hydraulic diffusivity tensor, ($\text{m}^2 \text{s}^{-1}$),
D_e	Energy dissipation for fluid flow in porous medium, (Pa^2),
D_L	Characteristic diffusivity at the field scale (m),
D_l	Characteristic diffusivity at the Darcy scale (m),
$E^{(n)}$	n th finite element,
$F^{(n)}$	n th face of a finite volume grid,
$G_{\alpha}^{(j)}$	Gravity term in spatially discretized flow equation for phase α at node j ,
H_{α}	Total potential head of fluid phase α , (m),
K_{sz}	Hydraulic conductivity of phase α at apparent saturation, (m s^{-1}),
K_{α}	Hydraulic conductivity of phase α , (m s^{-1}),
L	Characteristic length at the field scale, (m),
M_{α}	Mass density of fluid phase α with respect to the bulk volume of porous medium, $M_{\alpha} = \rho_{\alpha} \phi_{\alpha} S_{\alpha}$, (kg m^{-3}),
$Q_{\alpha}^{(\text{fc})}$	Total mass flux of phase α at a control volume face, (kg s^{-1}),
R_b	Characteristic dimension of a matrix block or inclusion, (m),
R_{gas}	Universal gas constant, ($\text{J}(\text{mol}^{\circ}\text{K})^{-1}$),
$S_{\text{a}}^{\text{trap}}$	Effective air saturation for a heterogeneous medium in trapped-air regime,
$S_{\text{w}}^{\text{trap}}$	Effective water saturation for a heterogeneous medium in trapped-air regime,
S_{α}	Saturation of fluid phase α ,
$S_{\text{e}\alpha}$	Normalized saturation of phase α ,
S_{α}^{max}	Maximum attainable saturation of phase α ,
S_{α}^{min}	Minimum attainable saturation of phase α ,

S_{rx}	Saturation of phase α at residual state,
T_x^m	Non-equilibrium mass flow rate between fractures and matrix, or between background material and inclusions, $(\text{kg}(\text{m}^3 \text{s})^{-1})$,
T_x^v	Non-equilibrium volumetric flow rate between fractures and matrix, or between background material and inclusions, (s^{-1}) ,
U	Domain of a representative elementary volume at the pore scale,
U_x	Part of a pore-scale representative elementary volume occupied by phase α ,
V_b	Matrix block domain,
$V^{(i)}$	i th finite volume,
W	Solution domain at the Darcy scale,
Z_k	Auxiliary parameter in the formula for inter-nodal permeability,
Z_s	Auxiliary parameter in the formula for relating parameters of the Brook-Corey and van Genuchten functions.

Calligraphic Latin letters:

$\mathcal{D}^{(j)}$	Spatial discretization operator for node j ,
$\mathcal{E}^{(j)}$	Set of finite elements sharing node j ,
\mathcal{H}	Relative air humidity,
\mathcal{M}_α	Mole mass of fluid phase α , (kg mol^{-1}) ,
$\mathcal{N}_{\text{elem}}^{(n)}$	Set of nodes belonging to n -th finite element,
$\mathcal{N}_{\text{nod}}^{(j)}$	Set of nodes connected to node j , including j ,
$\mathcal{O}(u)$	Order of magnitude of number u ,
\mathcal{R}_c	Dimensionless gravity–capillarity ratio,
\mathcal{R}_d	Dimensionless hydraulic diffusivity ratio between inclusions and background,
\mathcal{R}_k	Dimensionless permeability ratio between inclusions and background,
\mathcal{R}_t^l	Dimensionless time scale for Darcy-scale flow in porous material l ,
\mathcal{T}	Kelvin temperature, $(^\circ\text{K})$.

Lowercase Latin letters:

a_i	Length of the ellipsoidal inclusion in i th spatial direction, (m),
$a', a'_{10}, a'_{11}, a'_2$	Parameters in the Gasto et al. formula for the inter-nodal permeability,
$b', b'_{01}, b'_{02}, b'_1$	Parameters in the Gasto et al. formula for the inter-nodal permeability,
c', c'_0	Parameters in the Gasto et al. formula for the inter-nodal permeability,
d_i	Ellipsoid depolarisation coefficient in i th spatial direction,
f_z	Fractional flow function for fluid phase α ,
g	Magnitude of the gravitational acceleration vector, (m s^{-2}) ,

\mathbf{g}	Gravitational acceleration vector, (m s^{-2}),
h_α	Pressure head of fluid phase α , (Pa),
h_c	Capillary pressure head, (Pa),
h_e	Air-entry pressure head, (Pa),
h_g	Pressure head scaling parameter, (Pa),
$k_{ca}^{(ij)}$	Average relative water permeability for the capillary-driven flow between nodes i and j ,
$k_{gr}^{(ij)}$	Average relative water permeability for the gravity-driven flow between nodes i and j ,
$k_{int}^{(ij)}$	Integrated average relative water permeability between nodes i and j ,
k_{ii}^{cpl}	Cardwell and Parsons lower bound for the equivalent permeability in i th direction, $i = 1, 2, 3$, (m^2),
k_{ii}^{cpu}	Cardwell and Parsons upper bound for the equivalent permeability in i th direction, $i = 1, 2, 3$, (m^2),
$k_{rw}^{(el)}$	Average relative water permeability in a finite element,
k_{rw}^{fm}	Relative water permeability at the fracture–matrix interface,
$k_{rw}^{(ij)}$	Average relative water permeability between nodes i and j ,
$k_{r\alpha}$	Relative permeability of phase α ,
\mathbf{K}_s	Intrinsic permeability tensor, (m^2),
\mathbf{K}_s^{eq}	Equivalent intrinsic permeability tensor of heterogeneous medium, (m^2),
\mathbf{K}_s^{eff}	Effective intrinsic permeability tensor of a heterogeneous medium, (m^2),
\mathbf{k}_t	Total permeability tensor in fractional flow formulation, ($\text{m}^2(\text{Pa s})^{-1}$),
\mathbf{k}_α	Permeability tensor of phase α , (m^2),
\mathbf{K}_α^{eff}	Effective permeability tensor of a heterogeneous medium for phase α , (m^2),
\mathbf{K}_w^{high}	Effective water permeability tensor for a heterogeneous medium with highly permeable inclusions, (m^2),
\mathbf{K}_w^{low}	Effective water permeability tensor for a heterogeneous medium with weakly permeable inclusions, (m^2),
\mathbf{K}_w^{trap}	Effective water permeability tensor for a heterogeneous medium in trapped-air regime, (m^2),
l	Characteristic length at the Darcy scale, (m),
l_h	Characteristic dimension of Darcy-scale heterogeneities, (m),
l_v	Characteristic dimension of the averaging volume, (m),
m_g	Exponent in the van Genuchten capillary function,
n_b	Exponent in the Brooks–Corey capillary function,
n_g	Exponent in the van Genuchten capillary function,

\mathbf{n}_E	Unit vector normal to the boundary of a finite element,
\mathbf{n}_V	Unit vector normal to the boundary of a finite volume,
\mathbf{n}_W	Unit vector normal to the boundary of the porous domain,
\mathbf{n}	Unit vector normal to the interface separating two porous materials,
p_{atm}	Atmospheric pressure, (Pa),
p_c	Capillary pressure, (Pa),
p_e	Air-entry pressure, (Pa),
p_e^{drain}	Air-entry pressure during drainage, (Pa),
p_e^{wet}	Air-entry pressure during wetting, (Pa),
p_g	Capillary pressure scaling parameter in the Gardner and van Genuchten functions, (Pa),
p_α	Pressure in fluid phase α , (Pa),
p_α^{ref}	Reference pressure for fluid phase α , (Pa),
p_{glob}	Global pressure in the fractional flow formulation, (Pa),
\tilde{p}_i	Fluctuation of the fluid pressure for steady flow in i th spatial direction, (Pa),
q_{ev}	Cumulative evaporation flux at the soil surface, (m),
q_{inf}	Cumulative infiltration flux at the soil surface, (m),
r_b	Local spatial coordinate in a matrix block or inclusion, (m),
r_c	Radius of a capillary tube, (m),
r_{c1}, r_{c2}	Main curvature radii of the air–water interface, (m),
s_{abs}	Absolute error tolerance in the solution of nonlinear algebraic equations,
s_{rel}	Relative error tolerance in the solution of nonlinear algebraic equations,
t	Time, (s),
t_{dry}	Surface drying time in the evaporation simulation, (s),
t_{pond}	Surface ponding time in the infiltration simulation, (s),
u	Generic variable,
\mathbf{u}	Vector of unknown nodal values in the numerical solution,
v_L	Characteristic advective velocity at the field scale, (m s^{-1}),
v_l	Characteristic advective velocity at the Darcy scale, (m s^{-1}),
$v_{\text{st}}^{(ij)}$	Steady-state volumetric water flux between nodes i and j , (m s^{-1}),
v_w^{top}	Volumetric water flux at the soil surface, (m s^{-1}),
\mathbf{v}_α	Volumetric flux of fluid phase α with respect to the solid phase (Darcy velocity), (m s^{-1}),
\mathbf{v}_t	Total volumetric flux in the fractional flow formulation, (m s^{-1}),
w	Volumetric fraction of a porous material,
\mathbf{x}	Spatial coordinate vector, (m),
\mathbf{y}	Spatial coordinate vector associated with a periodic cell, (m),
z	Elevation above the reference level, (m).

Uppercase Greek letters:

Γ	Interface between two porous materials,
$\Delta x^{(ij)}$	Distance between nodes i and j , (m),
$\Delta x'$	Normalized distance between nodes,
Δt	Time step, (s),
Θ	Weighting coefficient in the time discretization scheme,
Λ^b	External surface of a matrix block,
$\Lambda^{(i+1/2)}$	Interface between nested matrix blocks i and $i + 1$ in MINC method,
Υ	Weighting function in the finite element method,
Φ_h	Flux potential with respect to the water pressure head, (m),
Φ_p	Flux potential with respect to the water pressure, (Pa),
$\Psi^{(i)}$	Shape function for node i in the finite element method,
$\Psi_e^{(i)}$	Element shape function for node i in the finite element method,
Ω	Domain of a periodic cell,
Ω'	Part of a periodic cell occupied by the background material,
Ω''	Part of a periodic cell occupied by the inclusions.

Lowercase Greek letters:

α_g	Inverse of the scaling pressure (or pressure head) in the capillary function, (Pa^{-1}) or (m^{-1}),
$\alpha_{p,q}$	Coefficients in the modified equation,
β_α	Relative compressibility coefficient for fluid phase α , (Pa^{-1}),
β^{fm}	Shape coefficient for the fracture–matrix transfer term,
β'	Parameter in the averaging formula for the inter-nodal permeability,
$\beta_{p,q}$	Coefficients in the modified equation,
γ^{fm}	Scaling coefficient for the fracture–matrix transfer term,
$\gamma_{p,q}$	Coefficients in the modified equation,
$\delta \mathbf{u}$	Increment of the vector of unknown values in the iterative solution procedure,
ε	Scale parameter,
ζ	Gravity coefficient, cosine of the angle between x axis and the gravity vector in one-dimensional problems,
ζ'	Modified gravity coefficient,
η_α	Exponent in the power-law relative permeability function for phase α ,
$\eta_1, \eta_2, \eta_3, \eta_4$	Exponents in the Mualem and Burdine relative permeability functions,
θ_α	Volumetric content of phase α ,

θ_{rx}	Volumetric content of phase at the residual state α ,
θ_{sz}	Volumetric content of phase α at the state of apparent saturation,
θ_a^{trap}	Effective volumetric air content for a heterogeneous medium in trapped-air regime,
θ_w^{trap}	Effective volumetric water content for a heterogeneous medium in trapped-air regime,
κ	Connectivity parameter in the Mualem and Burdine relative permeability functions,
λ_α	Mobility of phase α , $(\text{Pa s})^{-1}$,
μ_α	Dynamic viscosity coefficient of fluid phase α , (Pa s) ,
ξ	Local spatial coordinate in the finite element scheme,
π_i	Weighting coefficient in the generalized power average formula for the equivalent permeability in i th spatial direction,
ρ_α	Intrinsic mass density of fluid phase α , (kg m^{-3}) ,
ρ_α^{ref}	Reference intrinsic mass density of fluid phase α , (kg m^{-3}) ,
$\sigma_{\alpha\beta}$	Surface tension between phases α and β , (N m^{-1}) ,
ν	Small number used in numerical differentiation,
ϕ	Porosity,
χ_i	Auxiliary variable used to define the effective permeability in i th spatial direction,
	Wetting angle,
ω_k	Weighting parameter in the averaging scheme for fracture–matrix permeability,
ω_v	Weighting parameter in the averaging formula for the inter-nodal relative permeability,
ω_w	Weighting parameter in the averaging formula for the inter-nodal relative permeability.