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Numerical Analysis of Multiscale Problems

 Springer

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Preface

This book contains ten invited expository articles arising from the 91st LMS Durham Symposium on “Numerical Analysis of Multiscale Problems” which took place in the beautiful cathedral city of Durham in the UK from 5 to 15 July 2010. The Symposium was attended by 103 participants and was organised by Ivan Graham, Tom Hou and Rob Scheichl. The scientific programme highlighted novel research in theoretical numerical analysis and its applications to advances in areas such as oil reservoir modelling, high frequency scattering, data assimilation, waveguide modelling, uncertainty quantification, atomistic/continuum and polymer modelling. The selected articles in this book are written by some of the Symposium’s speakers and their collaborators, including all those speakers who gave short courses of three lectures each. The topics of the articles have been chosen to give a good overall representation of the scope of the Symposium and to provide a resource for researchers who would like to learn more about contemporary progress in this area. The five themes of the Symposium, and all the speakers who supported them are as follows.

Theme 1: *Numerical analysis for multiscale PDEs.* This theme was anchored by short courses of Todd Arbogast and Mitch Luskin. Todd Arbogast surveyed multiscale approximation for elliptic PDEs (in particular mixed methods), motivated by problems in oil reservoir modelling, while Mitch Luskin described the numerical analysis of problems arising in atomistic-continuum modelling in solid mechanics. There was also a strong analysis content to the short courses of Andrew Stuart, Markus Melenk and Christoph Schwab, described under the other themes below. Yalchin Efendiev discussed approximation of high contrast diffusion problems motivated by reservoir modelling, and his talk also had strong connections to Themes 4 and 5. Endre Süli presented a talk on the fundamental analysis of high-dimensional PDE models arising in polymer modelling, related strongly to Theme 3. Assyr Abdulle’s talk concerned adaptivity in the heterogeneous multiscale method, while the theme of atomistic/continuum modelling was continued in the talks of Ping Lin and Christoph Ortner. Richard Tsai discussed micro/macro-modelling

in porous media flow. Arieh Iserles talked on asymptotic-numerical multiscale expansions, which was also related to Theme 2. Radial basis function approximation applied to multiscale modelling problems in the geosciences was presented by Holger Wendland, while Chris Budd presented moving mesh adaptive PDE methods applied to problems in numerical weather forecasting.

Theme 2: *Multiscale wave propagation problems.* This theme was anchored by the short course of Markus Melenk, and lectures by Liliana Borcea (inverse problems for waves in random media), Olof Runborg (time domain problems in heterogeneous media) and Zhiming Chen (on wave propagation problems in infinite domains). Bjorn Engquist and Martin Gander both gave lectures on different aspects of robust solvers for high frequency wave problems (see Theme 4). A session was also devoted to asymptotic/numerical hybrid methods for wave problems and included talks of Peter Monk, Simon Chandler-Wilde, Timo Betcke and Euan Spence.

Theme 3: *Stochastic problems.* This central topic was anchored by short courses of Andrew Stuart (on the Bayesian approach to inverse problems and application to data assimilation) and Christoph Schwab (PDEs with random input data and related high dimensional parametrized PDEs). A new approach for numerical analysis of PDEs with lognormal permeability fields was discussed by Marcus Sarkis. The short course of Patrick Jenny focussed on applications, but also had strong resonance with this Theme. Talks were also given by Mike Giles (multilevel Monte Carlo), Oliver Ernst (generalised polynomial chaos), Catherine Powell (solvers for high dimensional discretizations of stochastic PDEs) and Frances Kuo (quasi-Monte Carlo methods), while Raul Tempone gave a lecture on discretisation of kinetic Monte Carlo models in computational chemistry and Tom Hou considered the reduced basis approach in a stochastic setting. Viet Ha Hoang discussed related random hyperbolic problems.

Theme 4: *Efficient solvers and computational aspects.* This theme was anchored by the lecture of Pater Bastian on the DUNE programming environment, sponsored by the Centre for Numerical Algorithms and Intelligent Software (Edinburgh/Heriot-Watt/Strathclyde), and the lecture of Bjorn Engquist on preconditioning in FEM and matrix compression in BEM, both for the Helmholtz equation. Martin Gander also gave a survey of high-frequency Helmholtz FEM solvers, with emphasis on optimised interface conditions. Frédéric Nataf and Ludmil Zikatanov discussed solvers for heterogeneous media problems, providing a different perspective on problems introduced by Yalchin Efendiev (in Theme 1). Catherine Powell discussed multigrid solvers for high-dimensional systems arising from pdes with random data (linked to Theme 3).

Theme 5: *Application areas.* This theme was anchored by the short course of Patrick Jenny who presented probability density function methods with applications to problems such as turbulent combustion and CO₂ sequestration, and was also linked to Theme 3. Lou Durlovsky discussed the state of the art in uncertainty quantification in reservoir modelling in the real industrial context. Solutions to multiscale differential equations modelling cell biology and molecular dynamics

were presented respectively by John King and Ben Leimkuhler. An embedded Industry Day included a number of external guests who joined with the participants of the full Symposium. Talks were given by Paul Childs (Schlumberger, seismic imaging), Anthony Baran (Met Office, scattering from ice crystals), Jill Ogilvy (BAE Systems, high frequency scattering), Roland Masson (Institut Français du Pétrole) and Seong Lee (Chevron) (both on multiphase flow), Tim Payne (Met Office, data assimilation in weather forecasting), Grigory Vilensky (UCL, PDE modelling of ultrasound in cancer treatment), Peter Jimack (Leeds, lubrication problems) and Andrew Cliffe (Nottingham, uncertainty quantification in waste management). An additional talk by Xiao-Hui Wu (ExxonMobil) on uncertainty quantification in reservoir simulation was presented later in the Symposium.

The scientific organisation was made straightforward by the excellent local support at Durham, especially the help of our close colleagues James Blowey and Max Jensen, and Fiona Giblin and the rest of the team in the Maths office, as well as John Bolton and John Parker. Postgraduate students John Chapman, Aretha Teckentrup, Ray Millward and Tatiana Kim operated the video camera.

Finally we would like to thank the UK Engineering and Physical Sciences Research Council, the London Mathematical Society and The Centre for Numerical Algorithms and Intelligent Software for the financial support which made the meeting possible.

More information about the 91st LMS Durham Symposium is available at:

<http://www.maths.dur.ac.uk/events/Meetings/LMS/2010/NAMP/>

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