Part IV
Gravity at Front Stage

When is the gravitational energy comparable with the Coulomb energy in a neutral system?

The gravitational energy U_G of an electrically neutral system of mass M grows with M as $U_G = aM^{5/3}$ (assuming that the density does not depend on M), while the Coulomb energy grows linearly with M as $U_C = bM$; the quantities a and b will be examined below. This difference in the rate of growing is due to the long range and always attractive character of the gravitational interaction versus the *effectively* short range character of the Coulomb interaction in an overall electrically neutral system. This effective short range property is due to the Coulomb interaction being either attractive, for opposite charged particles, or repulsive, for particles of the same charge. As a result in overall neutral systems positively and negatively charged particles combine to form *locally* neutral entities which attract each other through the short range residual van der Waals interaction. The gravitational energy will become equal to the Coulomb energy when the total mass will be equal to

$$M = (b/a)^{3/2} \tag{IV.1}$$

This characteristic exponent of 3/2 will appear very often in calculation involving gravity.

The gravitational self energy of a spherical body of mass M and radius R, according to Newton's law, equals to

$$U_G = -\frac{3x}{5}\frac{GM^2}{R} \tag{IV.2}$$

where x equals 1, if the mass density is homogeneous and isotropic; if the density is decreasing as the distance from the center is increasing, x is larger than 1. (For Earth, $x \approx 1.025$; see (11.11)). Taking into account that $M = (4\pi/3)R^3\,\rho_M$ and (9.3) we obtain that

$$a = -\frac{3x}{5}\frac{GA_W^{1/3}m_u^{1/3}}{a_B\bar{r}} \tag{IV.3}$$

A_w is the average atomic weight.

The Coulomb energy was obtained before in (10.10). Substituting in this formula, N_a from the relation $M = m_u A_W N_a = m_u N_v$, where N_v is the total number of nucleons, we find for b

$$b = -\frac{\gamma y}{\bar{r}} \frac{e^2}{a_B} \frac{1}{A_W m_u} \tag{IV.4}$$

where y is a correction numerical factor to take into account a non-uniform concentration of particles; y is expected to be a little larger than 1, but probably smaller than x. Combining (IV.1), (IV.3), (IV.4), and, $M = m_u N_v$, we find that, in an electrically neutral system, the number of nucleons N_v, which makes the gravitational energy equal to the Coulomb energy:

$$N_v = \left(\frac{5y}{3x}\right)^{3/2} \frac{\left(0.56\zeta^{4/3} + 0.9\zeta^2\right)^{3/2}}{A_W^2} \left(\frac{e^2}{G m_u^2}\right)^{3/2} \tag{IV.5}$$

where γ was set equal to $0.56\zeta^{4/3} + 0.9\zeta^2$ (see p. 85). The dimensionless ratio $e^2/Gm_u^2 \approx 1.2536 \times 10^{36}$ is the ratio of the strengths of the electromagnetic to the gravitational interactions (see Table 2.2, p. 10). The radius R corresponding to N_v is given by

$$R = \bar{r} a_B N_a^{1/3} = \bar{r} a_B N_a^{1/3} / A_W^{1/3}$$

If we choose $A_W \approx 40$ (as in Earth), $\zeta = 1$, $y/x = 1$, and $\bar{r} = 2.75$, we find $N_v = 3.3 \times 10^{51}$ and $R = 6.35 \times 10^6$ m versus $N_v = 3.596 \times 10^{51}$ and $R = 6.371 \times 10^6$ m for Earth. If we choose $A_W \approx 2$, $\zeta \approx 1$, $y/x \approx 1$, $\bar{r} \approx 1.6$, values close to those of Jupiter, we obtain $N_v \approx 1.33 \times 10^{54}$ and $R \approx 7.36 \times 10^7$ m versus $N_v \approx 1.143 \times 10^{54}$ and $R \approx 7.15 \times 10^7$ m for Jupiter. We conclude that for the Earth and for Jupiter the Coulomb energy is comparable to the gravitational energy. For the Sun the ratio U_G/U_C (assuming that its and A_W and ζ are as those of Jupiter) is equal to

$$U_G/U_C \approx (a/b) M_s^{2/3} \approx (M_S/M_J)^{2/3} \approx 100 \tag{IV.6}$$

where M_S, M_J are the masses of the Sun and the Jupiter respectively.