

Experimental Fluid Mechanics

For further volumes:

<http://www.springer.com/series/3837>

L.P. Yarin

The Pi-Theorem

Applications to Fluid Mechanics
and Heat and Mass Transfer

 Springer

L.P. Yarin
Technion-Israel Institute of Technology
Dept. of Mechanical Engineering
Technion City
32000 Haifa
Israel

ISBN 978-3-642-19564-8 e-ISBN 978-3-642-19565-5
DOI 10.1007/978-3-642-19565-5
Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2011944650

© Springer-Verlag Berlin Heidelberg 2012

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

*To the blessed memory of my parents Professor Peter Yarin
and Mrs. Leah Aranovich*

Preface

The book is devoted to the Buckingham Pi-theorem and its applications to various phenomena in nature and engineering. The accent is made on problems characteristic of heat and mass transfer in solid bodies, as well as in laminar and turbulent flows of liquids and gases. Such choice is not accidental. It is dictated by the requirements of modern technology and encompasses a vast majority of important problems related with drag and heat transfer experienced by solid bodies moving in viscous fluids. These problems involve the evaluation of temperature fields in media with constant and temperature-dependent thermal diffusivity, heat and mass transfer in boundary layers, pipe and jet flows, as well as thermal processes occurring in reactive media. In all these cases a uniform approach to the corresponding complex thermohydrodynamical problems is used. It is based on the direct application of the Pi-theorem to the analysis of two types of problems: those which admit a rigorous mathematical formulation, as well as those for which such formulation is unavailable. For the former problems our attention will be focused on the establishment of self-similarity which reduces the governing partial differential equations to the ordinary ones by means of the Pi-theorem, whereas for the latter problems the Pi-theorem will be used to reveal a set of the governing dimensionless groups. To a certain degree the choice of the problems is subjective. However, it allows the evaluation of the range of possible applications of the Pi-theorem and the peculiarities characteristic of the complex thermohydrodynamical processes in continuous media.

The book consists of nine chapters. They deal with the basics of the dimensional analysis, the application of the Pi-theorem to find self-similarities and reduce partial differential equations to the ordinary ones. Then, such interrelated topics as the drag force, laminar flows in channels, pipes and jets are covered in detail. The discussion also involves kindred heat and mass transfer in natural, forced and mixed convection and in situations with phase change and chemical reactions. Some problems of turbulence theory are also covered in the framework of the Pi-theorem. In addition to the in-depth exposition of the basic theory and the generic problems, a number of worked examples of problems related to the application of the Pi-theorem to different hydrodynamic, heat and mass transfer questions are presented in the end of each chapter. They can be interest to the engineering and physics students.

The book is intended to scientists and engineers interested in hydrodynamic and heat and mass transfer problems. It could also be useful to graduate students studying mechanical, civil and chemical engineering, as well as applied physics.

L.P. Yarin

Acknowledgment

I am especially grateful and deeply indebted to my son Professor Alexander Yarin for some special consultations related to the applications of the dimensional analysis to thermohydrodynamics problems, many insightful suggestions and discussions, as well as multiple comments on the contents of the book.

I am deeply obligated to my daughter Mrs. Elena Yarin and my granddaughter Miss Inna Yarin. Without their help this book would not have materialized.

Contents

1	The Overview and Scope of the Book	1
2	Basics of the Dimensional Analysis	3
2.1	Preliminary Remarks	3
2.2	Basic Definitions	3
2.2.1	Dimensional and Dimensionless Parameters	3
2.2.2	The Principle of Dimensional Homogeneity	7
2.3	Non-Dimensionalization of the Governing Equations	11
2.4	Dimensionless Groups	18
2.4.1	Characteristics of Dimensionless Groups	18
2.4.2	Similarity	21
2.5	The Pi-Theorem	23
2.5.1	General Remarks	23
2.5.2	Choice of the Governing Parameters	26
	Problems	30
	References	37
3	Application of the Pi-Theorem to Establish Self-Similarity and Reduce Partial Differential Equations to the Ordinary Ones	39
3.1	General Remarks	39
3.2	Flow over a Plane Wall Which Has Instantaneously Started Moving from Rest (the Stokes Problem)	44
3.3	Laminar Boundary Layer over a Flat Plate (the Blasius Problem)	47
3.4	Laminar Submerged Jet Issuing from a Thin Pipe (the Landau Problem)	51
3.5	Vorticity Diffusion in Viscous Fluid	54
3.6	Laminar Flow near a Rotating Disk (the Von Karman Problem)	55
3.7	Capillary Waves after a Weak Impact of a Tiny Object onto a Thin Liquid Film (the Yarin-Weiss Problem)	58
3.8	Propagation of Viscous-Gravity Currents over a Solid Horizontal Surface (the Huppert Problem)	60
3.9	Thermal Boundary Layer over a Flat Wall (the Pohlhausen Problem)	63

3.10 Diffusion Boundary Layer over a Flat Reactive Plate
 (the Levich Problem) 65
 Problems 67
 References 69

4 Drag Force Acting on a Body Moving in Viscous Fluid 71

4.1 Introductory Remarks 71

4.2 Drag Action on a Flat Plate 73

4.2.1 Motion with Constant Speed 73

4.2.2 Oscillatory Motion of a Plate Parallel to Itself 75

4.3 Drag Force Acting on Solid Particles 76

4.3.1 Drag Experienced by a Spherical Particle at Low,
 Moderate and High Reynolds Numbers 76

4.3.2 The Effect of Rotation 79

4.3.3 The Effect of Acceleration 80

4.3.4 The Effect of the Free Stream Turbulence 81

4.3.5 The Influence of the Particle-Fluid Temperature
 Difference 82

4.4 Drag of Irregular Particles 82

4.5 Drag of Deformable Particles 84

4.6 Drag of Bodies Partially Submerged in Liquid 86

4.7 Terminal Velocity of Small Spherical Particles Settling
 in Viscous Liquid (the Stokes Problem for a Sphere) 87

4.8 Sedimentation 90

4.8.1 Dimensionless Groups 90

4.8.2 Terminal Velocity of Heavy Grains 91

4.8.3 The Critical State of a Fluidized Bed 92

4.9 Thin Liquid Film on a Plate Withdrawn Vertically from
 a Pool Filled with Viscous Liquid (the Landau-Levich
 Problem of Dip Coating) 93

Problems 96

References 101

5 Laminar Flows in Channels and Pipes 103

5.1 Introductory Remarks 103

5.2 Flows in Straight Pipes of Circular Cross-Section 106

5.2.1 The Entrance Flow Region 106

5.2.2 Fully Developed Region of Laminar Flows
 in Smooth Pipes 109

5.2.3 Fully Developed Laminar and Turbulent
 Flows in Rough Pipe 109

5.3 Flows in Irregular Pipes and Ducts 111

5.4 Microchannel Flows 112

5.5 Non-Newtonian Flows 113

5.6	Flows in Curved Pipes	116
5.7	Unsteady Flows in Straight Pipes	120
	Problems	123
	References	129
6	Jet Flows	131
6.1	Introductory Remarks	131
6.2	The Far Field of Submerged Jets	136
6.3	The Dimensionless Groups of Jet Flows	139
6.4	Plane Laminar Submerged Jet	141
6.5	Laminar Wake of a Blunt Solid Body	143
6.6	Wall Jets over Plane and Curved Surfaces	146
6.7	Buoyant Jets (Plumes)	149
	Problems	154
	References	156
7	Heat and Mass Transfer	159
7.1	Introductory Remarks	159
7.2	Conductive Heat and Mass Transfer	160
7.2.1	Temperature Field Induced by Plane Instantaneous Thermal Source	160
7.2.2	Temperature Field Induced by a Pointwise Instantaneous Thermal Source	161
7.2.3	Evolution of Temperature Field in Medium with Temperature-Dependent Thermal Diffusivity (The Zel'dovich-Kompaneyets Problem)	162
7.3	Heat and Mass Transfer Under Conditions of Forced Convection ..	165
7.3.1	Heat Transfer from a Hot Body Immersed in Fluid Flow	165
7.3.2	The Effect of Particle Rotation	169
7.3.3	The Effect of the Free Stream Turbulence	171
7.3.4	The Effect of Energy Dissipation	173
7.3.5	The Effect of Velocity Gradient	174
7.3.6	Mass Transfer to Solid Particles and Drops Immersed in Fluid Flow	176
7.4	Heat and Mass Transfer in Channel and Pipe Flows	178
7.4.1	Couette Flow	178
7.4.2	The Entrance Region of a Pipe	180
7.4.3	Fully Developed Flow	181
7.5	Thermal Characteristics of Laminar Jets	183
7.6	Heat and Mass Transfer in Natural Convection	186
7.6.1	Heat Transfer from a Spherical Particle Under the Conditions of Natural Convection	186
7.6.2	Heat Transfer from Spinning Particle Under the Condition of Mixed Convection	187

- 7.6.3 Mass Transfer from a Spherical Particle Under the Conditions of Natural and Mixed Convection 189
- 7.6.4 Heat Transfer From a Vertical Heated Wall 190
- 7.6.5 Mass Transfer to a Vertical Reactive Plate Under the Conditions of Natural Convection 193
- 7.7 Heat Transfer From a Flat Plate in a Uniform Stream of Viscous, High Speed Gas 195
- 7.8 Heat Transfer Related to Phase Change 199
 - 7.8.1 Heat Transfer Due to Condensation of Saturated Vapor on a Vertical Wall 199
 - 7.8.2 Freezing of a Pure Liquid (The Stefan Problem) 202
- Problems 205
- References 209

- 8 Turbulence** 211
 - 8.1 Introductory Remarks 211
 - 8.2 Decay of Isotropic Turbulence 215
 - 8.3 Turbulent Near-Wall Flows 217
 - 8.3.1 Plane-Parallel Flows 217
 - 8.3.2 Pipe Flows 220
 - 8.3.3 Turbulent Boundary Layer 221
 - 8.4 Friction in Pipes and Ducts 222
 - 8.4.1 Friction in Smooth Pipes 222
 - 8.4.2 Friction in Rough Pipes 223
 - 8.5 Turbulent Jets 224
 - 8.5.1 Eddy Viscosity and Thermal Conductivity 224
 - 8.5.2 Plane and Axisymmetric Turbulent Jets 229
 - 8.5.3 Inhomogeneous Turbulent Jets 232
 - 8.5.4 Co-flowing Jets 238
 - 8.5.5 Turbulent Jets in Crossflow 245
 - 8.5.6 Turbulent Wall Jets 248
 - 8.5.7 Impinging Turbulent Jet 252
 - Problems 254
 - References 258

- 9 Combustion Processes** 261
 - 9.1 Introductory Remarks 261
 - 9.2 Thermal Explosion 265
 - 9.3 Combustion Waves 268
 - 9.4 Combustion of Non-premixed Gases 271
 - 9.5 Diffusion Flame in the Mixing Layer of Parallel Streams of Gaseous Fuel and Oxidizer 274

9.6 Gas Torches	280
9.7 Immersed Flames	288
Problems	294
References	296
Author Index	297
Subject Index	303

Nomenclature

Chapter 2:

Ar	Archimedes number
Bi	Biot number
Bo	Bond number
Br	Brinkman number
C	Speed of sound
C_d	Drag coefficient
c	Concentration
Ca	Capillary number
c_p	Specific heat
Da	Damkohler number
Da	Darcy number
De	Dean number
De	Deborah number
D	Diffusivity
D_*	Permeability coefficient of porous medium
Ec	Eckert number
Ek	Ekman number
Eu	Euler number
F_d	Drag force
Fr	Froude number
F_g	Gravity force
f	Frequency
g	Gravity acceleration
Gr	Grashof number
h	Heat transfer coefficient, or enthalpy
h_m	Mass transfer coefficient
Ja	Jacob number
k	Thermal conductivity
k_B	Boltzmann's constant
Kn	Knudsen number
Ku	Kutateladze number
L	Characteristic length scale
L_h	Height of liquid layer

l	Length of a pipe
Le	Lewis number
m	Mass of a particle
M	Mach number
Nu	Nusselt number
P	Pressure
ΔP	Pressure drop
Pe	Peclet number
Pe_d	Peclet number (for diffusion)
Pr	Prandtl number
Q_v	Volumetric flow rate
q	Heat of reaction
R	Gas constant, or radius of curvature
r	Cross-sectional radius of a pipe
Ra	Rayleigh number
Re	Reynolds number
Ri	Richardson number
Ro	Rossby number
r_v	Latent heat of vaporization
Sc	Schmidt number
Se	Semenov number
Sh	Sherwood number
St	Stanton number
St	Strouhal number
Ta	Taylor number
T	Temperature, or torque
ΔT	Temperature difference
t	Time
t_r	Relaxation time
t_0	Observation time
u	Particle velocity
v	Velocity vector with components u , v and w in projections to the Cartesian axes x , y and z
v	Specific volume
V_m	Mass flow rate
W	Rate of chemical reaction rate, or power (Watt)
We	Weber number
x, y, z	Cartesian coordinates

Greek Symbols

β	Coefficient of bulk expansion
γ	Ratio of the specific heat at constant pressure to the specific heat at constant volume (the adiabatic index)
δ	Boundary layer thickness
δ_T	Thermal boundary layer thickness
θ	Dimensionless temperature
Λ	Angle between the axis of Earth rotation and the direction of fluid motion
λ	Mean free path
μ	Viscosity
ν	Kinematic viscosity
ρ	Density
σ	Surface tension

τ	Time
τ_r	Relaxation time
τ_0	Observation time
ϕ	Dissipation function
ω	Angular velocity
ω_e	Angular velocity of Earth's rotation

Subscripts

f	Fluid
v	Vapor
w	Wall
∞	Undisturbed fluid at infinity

Chapter 3:

α	Thermal diffusivity
D	Diffusivity
g	Gravity acceleration
h	Thickness of liquid layer
J	Total momentum flux in jet
j	Diffusion flux
P	Pressure
Pr	Prandtl number
Q	Source strength
r	Radial coordinate
r, θ, φ	Spherical coordinates
S	Surface or surface area
Sc	Schmidt number
t	Time
U	Plate or flow velocity in x direction
u	Fluid velocity
v	Velocity vector with components v_r , v_θ , and v_φ in spherical coordinate system

Greek Symbols

α	Thermal diffusivity, exponent
Γ	Strength of an infinitely thin vortex line
δ	Thickness of the boundary layer
η	Dimensionless variable
ϑ	Dimensionless temperature
ν	Kinematic viscosity
ρ	Density
σ	Surface tension
τ	Shear stress
φ	Polar angle, dimensionless function
Ω	Vorticity component normal to the flow plane, or angular velocity

Subscript

∞ Undisturbed fluid

Chapter 4:

A_c	Acceleration parameter
c_d	Drag coefficient
c_l	Lift coefficient
d	Diameter
f_d	Drag force
f_l	Lift force
g	Gravity acceleration
l	Scale of turbulence, length of plate
P	Pressure
Q	Volumetric flow rate
R	Radius
T	Dimensionless turbulence intensity
\overline{u}	Root-mean square of turbulent fluctuations
u, v, w	Velocity components
\mathbf{v}	Velocity vector
Fr	Froude number
Re	Reynolds number
We	Weber number

Greek Symbols

α	Angle
μ	Viscosity
ν	Kinematic viscosity
ρ	Density
σ	Surface tension
τ	Shear stress at the wall
γ	Dimensionless angular velocity
ω	Angular velocity

Subscripts

d	Drag
l	Lift
p	Particle
∞	Ambient

Chapter 5:

d	Diameter
F_l	Inertial force
F_c	Centrifugal force

F_v	Friction force
Fo	Fouier number
k	Dean number, or roughness
K	Modified Dean number
l	The entrance length of pipe
l_*	Characteristic length of pipe
P	Pressure
ΔP	Pressure drop
Po	Poiseuille number
Q	Volumetric flow rate
R	Radius of curvature of a torus
Re	Reynolds number
r_0	Cross-sectional radius of a pipe
t	Time
u, v, w	Velocity components
u_0	Initial velocity
u_{\max}	Maximum velocity
\mathbf{v}	Velocity vector
w_0	Mean velocity
x, y, z	Cartesian coordinates
r, θ, x	Cylindrical coordinates

Greek Symbols

α	Large semi-axis of an ellipse
β	Small semi-axis of an ellipse
γ	Shear rate
δ	Ratio of pipe radius to its curvature
λ	Friction factor
μ	Viscosity
μ_0	Viscosity of Bingham fluid
ν	Kinematic viscosity
Π	Bingham number
ρ	Density
τ	Shear stress, geometric torsion
τ_0	Yield stress

Chapter 6:

h	Enthalpy
I_x	Kinematic momentum flux
J_x	Momentum flux
k	Thermal conductivity
M_x	Total moment-of-momentum flux
P	Pressure
Pr	Prandtl number
Re_δ	Local Reynolds number
T	Temperature
u	Longitudinal velocity component
v	Transversal velocity component

Greek Symbols

β	Thermal expansion coefficient
δ	Jet thickness
μ	Viscosity
ν	Kinematic viscosity
ϑ	Excessive temperature
ρ	Density

Subscripts

∞	Undisturbed fluid
m	Jet axis

Chapter 7:

c	Specific heat capacity, concentration
c_p	Specific heat at constant pressure
c_v	Specific heat at constant volume
D	Diffusivity
d	Diameter
E	Pointwise energy release
g	Gravity acceleration
H	Channel height
h	Heat transfer coefficient, rate of heat transfer, enthalpy
j	Mechanical equivalent of heat
k	Thermal conductivity
k_B	Boltzmann's constant
l	Turbulence scale
P	Pressure
Q	Strength of thermal source
q	Heat flux
q_l	Latent heat of freezing
r	Radius
T	Temperature
T_u	Turbulence intensity
v, u	Velocity
\tilde{v}	Velocity fluctuation
Ec	Eckert number
Gr	Grashof number
M	Mach number
Nu	Nusselt number
Pe	Peclet number
Pr	Prandtl number
Ra	Rayleigh number
Re	Reynolds number
Re_ω	Rotational Reynolds number
Sh	Sherwood number
St	Stephan number

Greek Symbols

α	Thermal diffusivity
β	Thermal expansion coefficient
γ	Ratio of specific heat at constant pressure to specific heat at constant volume (the adiabatic index)
δ	Delta function; boundary layer thickness
χ	Radiant thermal diffusivity
μ	Viscosity
ν	Kinematic viscosity
ρ	Density

Subscripts

en	Entrance
f	Front of thermal wave
P	Pressure
T	Thermal
W	Wall
∞	Undisturbed flow

Chapter 8:

A	Cross-sectional area of a jet
C	Concentration
d_0	Nozzle diameter
d_c	Nozzle width
Fr	Froude number
G_x	Total mass flux
H	Distance between the nozzle exit and the unperturbed liquid surface
h_c	Cavity depth
J_0	The exit kinematic momentum flux
J_x	Total momentum flux
l	Characteristic length
Pr	Prandtl number
P	Pressure
Re	Reynolds number
T	Temperature
u,v	Velocity components
u_m	Centerline velocity
We	Weber number

Greek Symbols

α_T	Eddy thermal diffusivity
δ	Jet half-width
η	Dimensionless variable
μ	Viscosity

μ_T	Eddy viscosity
ν	Kinematic viscosity
ν_T	Eddy kinematic viscosity
ρ	Density
σ	Surface tension

Subscripts

G	Gas
L	Liquid

Chapter 9:

c	Reactant concentration
c_p	Specific heat
D	Diffusivity
E	Activation energy
h	Enthalpy
k	Chemical reaction constant; thermal conductivity
k_0	Pre-exponential
Le	Lewis number
l_f	Flame length
P	Pressure
Pe	Peclet number
Q_1	Heat release
Q_2	Heat losses
q	Heat of reaction
R	The universal gas constant
Re	Reynolds number
T	Temperature
u_f	Speed of combustion wave
u_0	Speed of reactive mixture at the nozzle exit
W	Rate of chemical reaction
W_j	Rate of conversion of the j -th species
z	Pre-exponential

Greek Symbols

α	Thermal diffusivity
δ	Frank-Kamenetskii parameter
μ	Viscosity
ν	Kinematic viscosity
ρ	Density
τ_k	Characteristic kinetic time
τ_D	Characteristic diffusion time
Ω	Stoichiometric oxidizer-to-fuel mass ratio

Subscripts

<i>f</i>	Fuel
<i>o</i>	Oxidizer
<i>m</i>	Maximum; axis
0	Initial state
*	Gas-liquid interface

