Experimental Fluid Mechanics

For further volumes:
http://www.springer.com/series/3837

L.P. Yarin

The Pi-Theorem

Applications to Fluid Mechanics and Heat and Mass Transfer

Springer

L.P. Yarin
Technion-Israel Institute of Technology
Dept. of Mechanical Engineering
Technion City
32000 Haifa
Israel

ISBN 978-3-642-19564-8 e-ISBN 978-3-642-19565-5
DOI 10.1007/978-3-642-19565-5
Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2011944650

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

To the blessed memory of my parents Professor Peter Yarin and Mrs. Leah Aranovich

Preface

The book is devoted to the Buckingham Pi-theorem and its applications to various phenomena in nature and engineering. The accent is made on problems characteristic of heat and mass transfer in solid bodies, as well as in laminar and turbulent flows of liquids and gases. Such choice is not accidental. It is dictated by the requirements of modern technology and encompasses a vast majority of important problems related with drag and heat transfer experienced by solid bodies moving in viscous fluids. These problems involve the evaluation of temperature fields in media with constant and temperature-dependent thermal diffusivity, heat and mass transfer in boundary layers, pipe and jet flows, as well as thermal processes occurring in reactive media. In all these cases a uniform approach to the corresponding complex thermohydrodynamical problems is used. It is based on the direct application of the Pi-theorem to the analysis of two types of problems: those which admit a rigorous mathematical formulation, as well as those for which such formulation is unavailable. For the former problems our attention will be focused on the establishment of self-similarity which reduces the governing partial differential equations to the ordinary ones by means of the Pi-theorem, whereas for the latter problems the Pi-theorem will be used to reveal a set of the governing dimensionless groups. To a certain degree the choice of the problems is subjective. However, it allows the evaluation of the range of possible applications of the Pi-theorem and the peculiarities characteristic of the complex thermohydrodynamical processes in continuous media.

The book consists of nine chapters. They deal with the basics of the dimensional analysis, the application of the Pi-theorem to find self-similarities and reduce partial differential equations to the ordinary ones. Then, such interrelated topics as the drag force, laminar flows in channels, pipes and jets are covered in detail. The discussion also involves kindred heat and mass transfer in natural, forced and mixed convection and in situations with phase change and chemical reactions. Some problems of turbulence theory are also covered in the framework of the Pi-theorem. In addition to the in-depth exposition of the basic theory and the generic problems, a number of worked examples of problems related to the application of the Pi-theorem to different hydrodynamic, heat and mass transfer questions are presented in the end of each chapter. They can be interest to the engineering and physics students.

The book is intended to scientists and engineers interested in hydrodynamic and heat and mass transfer problems. It could also be useful to graduate students studying mechanical, civil and chemical engineering, as well as applied physics.

L.P. Yarin

Acknowledgment

I am especially grateful and deeply indebted to my son Professor Alexander Yarin for some special consultations related to the applications of the dimensional analysis to thermohydrodynamics problems, many insightful suggestions and discussions, as well as multiple comments on the contents of the book.

I am deeply obligated to my daughter Mrs. Elena Yarin and my granddaughter Miss Inna Yarin. Without their help this book would not have materialized.

Contents

Nomenclature

Chapter 2:

Ar	Archimedes number
Bi	Biot number
Bo	Bond number
Br	Brinkman number
C	Speed of sound
C_d	Drag coefficient
c	Concentration
Ca	Capillary number
c_P	Specific heat
Da	Damkohler number
Da	Darcy number
De	Dean number
De	Deborah number
D	Diffusivity
D_*	Permeability coefficient of porous medium
Ec	Eckert number
Ek	Ekman number
Eu	Euler number
F_d	Drag force
Fr	Froude number
F_g	Gravity force
f	Frequency
g	Gravity acceleration
Gr	Grashof number
h	Heat transfer coefficient, or enthalpy
h_m	Mass transfer coefficient
Ja	Jacob number
k	Thermal conductivity
k_B	Boltzmann's constant
Kn	Knudsen number
Ku	Kutateladze number
L	Characteristic length scale
L_h	Height of liquid layer

l Length of a pipe
Le Lewis number
m Mass of a particle
M Mach number
Nu Nusselt number
P Pressure
ΔP Pressure drop
Pe Peclet number
Pe_d Peclet number (for diffusion)
Pr Prandtl number
Q_v Volumetric flow rate
q Heat of reaction
R Gas constant, or radius of curvature
r Cross-sectional radius of a pipe
Ra Rayleigh number
Re Reynolds number
Ri Richardson number
Ro Rossby number
r_v Latent heat of vaporization
Sc Schmidt number
Se Semenov number
Sh Sherwood number
St Stanton number
St Strouhal number
Ta Taylor number
T Temperature, or torque
ΔT Temperature difference
t Time
t_r Relaxation time
t_0 Observation time
u Particle velocity
v Velocity vector with components u, v and w in projections to the Cartesian axes x, y and z
v Specific volume
V_m Mass flow rate
W Rate of chemical reaction rate, or power (Watt)
We Weber number
x, y, z Cartesian coordinates

Greek Symbols

β Coefficient of bulk expansion
γ Ratio of the specific heat at constant pressure to the specific heat at constant volume (the adiabatic index)
δ Boundary layer thickness
δ_T Thermal boundary layer thickness
θ Dimensionless temperature
Λ Angle between the axis of Earth rotation and the direction of fluid motion
λ Mean free path
μ Viscosity
ν Kinematic viscosity
ρ Density
σ Surface tension

τ	Time
τ_r	Relaxation time
τ_0	Observation time
ϕ	Dissipation function
ω	Angular velocity
ω_e	Angular velocity of Earth's rotation

Subscripts

f	Fluid
v	Vapor
w	Wall
∞	Undisturbed fluid at infinity

Chapter 3:

α	Thermal diffusivity
D	Diffusivity
g	Gravity acceleration
h	Thickness of liquid layer
J	Total momentum flux in jet
j	Diffusion flux
P	Pressure
Pr	Prandtl number
Q	Source strenght
r	Radial coordinate
$r,\ \theta,\ \varphi$	Spherical coordinates
S	Surface or surface area
Sc	Schmidt number
t	Time
U	Plate or flow velocity in x direction
u	Fluid velocity
v	Velocity vector with components $v_r,\ v_{\theta,}$ and v_φ in spherical coordinate system

Greek Symbols

α	Thermal diffusivity, exponent
Γ	Strength of an infinitely thin vortex line
δ	Thickness of the boundary layer
η	Dimensionless variable
ϑ	Dimensionless temperature
ν	Kinematic viscosity
ρ	Density
σ	Surface tension
τ	Shear stress
φ	Polar angle, dimensionless function
$\mathbf{\Omega}$	Vorticity component normal to the flow plane, or angular velocity

Subscript

∞	Undisturbed fluid

Chapter 4:

Ac	Acceleration parameter
c_d	Drag coefficient
c_l	Lift coefficient
d	Diameter
f_d	Drag force
f_l	Lift force
g	Gravity acceleration
l	Scale of turbulence, length of plate
P	Pressure
Q	Volumetric flow rate
R	Radius
T	Dimensionless turbulence intensity
$\overline{u'}$	Root-mean square of turbulent fluctuations
u, v, w	Velocity components
$\mathbf{v}$	Velocity vector
Fr	Froud number
Re	Reynolds number
We	Weber number

Greak Symbols

α	Angle
μ	Viscosity
v	Kinematic viscosity
ρ	Density
σ	Surface tension
τ	Shear stress at the wall
γ	Dimensionless angular velocity
ω	Angular velocity

Subscripts

d	Drag
l	Lift
p	Particle
∞	Ambient

Chapter 5:

d	Diameter
F_I	Inertial force
F_c	Centrifugal force

F_v	Friction force
Fo	Fouier number
k	Dean number, or roughness
K	Modified Dean number
l	The entrance length of pipe
l_*	Characteristic length of pipe
P	Pressure
ΔP	Pressure drop
Po	Poiseuille number
Q	Volumetric flow rate
R	Radius of curvature of a torus
Re	Reynolds number
r_0	Cross-sectional radius of a pipe
t	Time
u, v, w	Velocity components
u_0	Initial velocity
$u_{\max}$	Maximum velocity
$\mathbf{v}$	Velocity vector
w_0	Mean velocity
x, y, z	Cartesian coordinates
r, θ, x	Cylindrical coordinates

Greek Symbols

α	Large semi-axis of an ellipse
β	Small semi-axis of an ellipse
γ	Shear rate
δ	Ratio of pipe radius to its curvature
λ	Friction factor
μ	Viscosity
μ_0	Viscosity of Binham fluid
v	Kinematic viscosity
$\prod$	Bingham number
ρ	Density
τ	Shear stress, geometric torsion
τ_0	Yield stress

Chapter 6:

h	Enthalpy
I_x	Kinematic momentum flux
J_x	Momentum flux
k	Thermal conductivity
M_x	Total moment-of-momentum flux
P	Pressure
Pr	Prandtl number
Re_δ	Local Reynolds number
T	Temperature
u	Longitudinal velocity component
v	Transversal velocity component

Greek Symbols

β Thermal expansion coefficient
δ Jet thickness
μ Viscosity
v Kinematic viscosity
ϑ Excessive temperature
ρ Density

Subscripts

∞ Undisturbed fluid
m Jet axis

Chapter 7:

c Specific heat capacity, concentration
c_P Specific heat at constant pressure
c_v Specific heat at constant volume
D Diffusivity
d Diameter
E Pointwise energy release
g Gravity acceleration
H Channel height
h Heat transfer coefficient, rate of heat transfer, enthalpy
j Mechanical equivalent of heat
k Thermal conductivity
k_B Boltzmann's constant
l Turbulence scale
P Pressure
Q Strength of thermal source
q Heat flux
q_l Latent heat of freezing
r Radius
T Temperature
T_u Turbulence intensity
v, u Velocity
$\widetilde{v'}$ Velocity fluctuation
Ec Eckert number
Gr Grashof number
M Mach number
Nu Nusselt number
Pe Peclet number
$\Pr$ Prandtl number
Ra Rayleigh number
Re Reynolds number
Re_ω Rotational Reynolds number
Sh Sherwood number
St Stephan number

Greek Symbols

α Thermal diffusivity
β Thermal expansion coefficient
γ Ratio of specific heat at constant pressure to specific heat at constant volume (the adiabatic index)
δ Delta function; boundary layer thickness
χ Radiant thermal diffusivity
μ Viscosity
ν Kinematic viscosity
ρ Density

Subscripts

en Entrance
f Front of thermal wave
P Pressure
T Thermal
W Wall
∞ Undisturbed flow

Chapter 8:

A Cross-sectional area of a jet
C Concentration
d_0 Nozzle diameter
d_c Nozzle width
Fr Froude number
G_x Total mass flux
H Distance between the nozzle exit and the unperturbed liquid surface
h_c Cavity depth
I_0 The exit kinematic momentum flux
J_x Total momentum flux
l Characteristic length
Pr Prandtl number
P Pressure
Re Reynolds number
T Temperature
u,v Velocity components
u_m Centerline velocity
We Weber number

Greek Symbols

α_T Eddy thermal diffusivity
δ Jet half-width
η Dimensionless variable
μ Viscosity

μ_T Eddy viscosity
ν Kinematic viscosity
ν_T Eddy kinematic viscosity
ρ Density
σ Surface tension

Subscripts

G Gas
L Liquid

Chapter 9:

c Reactant concentration
c_P Specific heat
D Diffusivity
E Activation energy
h Enthalpy
k Chemical reaction constant; thermal conductivity
k_0 Pre-exponential
Le Lewis number
l_f Flame length
P Pressure
Pe Peclet number
Q_1 Heat release
Q_2 Heat losses
q Heat of reaction
R The universal gas constant
Re Reynolds number
T Temperature
u_f Speed of combustion wave
u_0 Speed of reactive mixture at the nozzle exit
W Rate of chemical reaction
W_j Rate of conversion of the j-th species
z Pre-exponential

Greek Symbols

α Thermal diffusivity
δ Frank-Kamenetskii parameter
μ Viscosity
ν Kinematic viscosity
ρ Density
τ_k Characteristic kinetic time
τ_D Characteristic diffusion time
Ω Stoichiometric oxidizer-to-fuel mass ratio

Subscripts

f	Fuel
o	Oxidizer
m	Maximum; axis
0	Initial state
$*$	Gas-liquid interface