



# Intelligent Systems Reference Library, Volume 9

## Editors-in-Chief

Prof. Janusz Kacprzyk  
Systems Research Institute  
Polish Academy of Sciences  
ul. Newelska 6  
01-447 Warsaw  
Poland  
*E-mail:* kacprzyk@ibspan.waw.pl

Prof. Lakhmi C. Jain  
University of South Australia  
Adelaide  
Mawson Lakes Campus  
South Australia 5095  
Australia  
*E-mail:* Lakhmi.jain@unisa.edu.au

---

Further volumes of this series can be found on our homepage: [springer.com](http://springer.com)

Vol. 1. Christine L. Mumford and Lakhmi C. Jain (Eds.)  
*Computational Intelligence: Collaboration, Fusion  
and Emergence*, 2009  
ISBN 978-3-642-01798-8

Vol. 2. Yuehui Chen and Ajith Abraham  
*Tree-Structure Based Hybrid  
Computational Intelligence*, 2009  
ISBN 978-3-642-04738-1

Vol. 3. Anthony Finn and Steve Scheduling  
*Developments and Challenges for  
Autonomous Unmanned Vehicles*, 2010  
ISBN 978-3-642-10703-0

Vol. 4. Lakhmi C. Jain and Chee Peng Lim (Eds.)  
*Handbook on Decision Making: Techniques  
and Applications*, 2010  
ISBN 978-3-642-13638-2

Vol. 5. George A. Anastassiou  
*Intelligent Mathematics: Computational Analysis*, 2010  
ISBN 978-3-642-17097-3

Vol. 6. Ludmila Dymowa  
*Soft Computing in Economics and Finance*, 2011  
ISBN 978-3-642-17718-7

Vol. 7. Gerasimos G. Rigatos  
*Modelling and Control for Intelligent Industrial Systems*, 2011  
ISBN 978-3-642-17874-0

Vol. 8. Edward H.Y. Lim, James N.K. Liu, and Raymond S.T. Lee  
*Knowledge Seeker – Ontology Modelling for Information  
Search and Management*, 2011  
ISBN 978-3-642-17915-0

Vol. 9. Menahem Friedman and Abraham Kandel  
*Calculus Light*, 2011  
ISBN 978-3-642-17847-4

Menahem Friedman and Abraham Kandel

# Calculus Light

Prof. Menahem Friedman  
Ben Gurion University of the Negev  
Beer-Sheva 84105  
Israel  
E-mail: mlfrid@netvision.net.il

Prof. Abraham Kandel  
University of South Florida  
4202 E. Fowler Ave. ENB 118  
Tampa  
Florida 33620  
USA  
E-mail: kandel@babbage.csee.usf.edu

ISBN 978-3-642-17847-4

e-ISBN 978-3-642-17848-1

DOI 10.1007/978-3-642-17848-1

Intelligent Systems Reference Library

ISSN 1868-4394

© 2011 Springer-Verlag Berlin Heidelberg

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under the German Copyright Law.

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

*Typeset & Cover Design:* Scientific Publishing Services Pvt. Ltd., Chennai, India.

Printed on acid-free paper

9 8 7 6 5 4 3 2 1

springer.com

*To our grandchildren*

*Gal, Jonathan and Tomer*

*Kfeer, Maya and Riley*

*with love*

## Preface

The first question we may face is: “Another Calculus book?” We feel that the answer is quite simple. As long as students find mathematics and particularly calculus a scary subject, as long as the failure rate in mathematics is higher than in all other subjects, except maybe among students who take it as a major in College and as long as a large majority of the people mistakenly believe that only geniuses can learn and understand mathematics and particularly analysis, there will always be room for a new book of Calculus. We call it Calculus Light.

This book is designed for a one semester course in "light" calculus, and meant to be used by undergraduate students without a deep mathematical background who do not major in mathematics or electrical engineering but are interested in areas such as biology or management information systems. The book's level is suitable for students who previously studied some elementary course in general mathematics. Knowledge of basic terminology in linear algebra, geometry and trigonometry is advantageous but not necessary.

In writing this manuscript we faced two dilemmas. The first, what subjects should be included and to what extent. We felt that a modern basic book about calculus dealing with traditional material of single variable is our goal. The introduction of such topics and their application in solving real world problems demonstrate the necessity and applicability of calculus in practically most walks of life.

Our second dilemma was how far we ought to pursue proofs and accuracy. We provided rigorous proofs whenever it was felt that the readers would benefit either by better understanding the specific subject, or by developing their own creativity. At numerous times, when we believed that the readers were ready, we left them to complete part or all of the proof. Certain proofs were beyond the scope of this book and were omitted. However, it was most important for us never to mix intuition and heuristic ideas with rigorous arguments.

We start this book with a historical background. Every scientific achievement involves people and is therefore characterized by victories and disappointments, cooperation and intrigues, hope and heartbreak. All of these elements exist in the story behind calculus and when you add the time dimension – over 2400 years since it all started, you actually get a saga. We hope the reader enjoys reading the first chapter as much as we enjoyed the writing.

In chapters 2-7 we present the topic of single variable calculus and these chapters should be studied in sequential order. The next two chapters provide basic theory and applications of Fourier series and elementary numerical methods. They are expected to motivate the student who is interested in applications and practicality.

The final chapter contains several special topics and combines beauty - the proof that  $e$  is irrational, and practicality - the theory of Lagrange multipliers introduced with a short introduction to multi-variable calculus.

Each chapter is divided into sections and at the end of almost every section, variety of problems is given. The problems are usually arranged according to the order of the respective topics in the text. Each topic is followed by examples, simple and complex alike, solved in detail and graphs are presented whenever they are needed. In addition we provide answers to selected problems.

It should be noted that the content of this book was successfully tested on many classes of students for over thirty years. We thank many of them for their constructive suggestions, endless reviews and enormous support.

Tampa, FL 2010

M. Friedman  
A. Kandel

# Contents

<b>1</b>	<b>Historical Background.....</b>	<b>1</b>
1.1	Prelude to Calculus or the Terror of the ‘Infinite’ .....	1
1.2	Calculus – Where Do We Start? .....	2
1.3	The Countdown .....	5
1.4	The Birth of Calculus.....	6
1.5	The Priority Dispute .....	8
<b>2</b>	<b>The Number System.....</b>	<b>11</b>
2.1	Basic Concepts about Sets .....	11
2.2	The Natural Numbers .....	14
2.3	Integers and Rational Numbers.....	17
2.3.1	Integers .....	18
2.3.2	Rational Numbers .....	19
2.4	Real Numbers .....	22
2.5	Additional Properties of the Real Numbers .....	30
<b>3</b>	<b>Functions, Sequences and Limits.....</b>	<b>37</b>
3.1	Introduction .....	37
3.2	Functions .....	39
3.3	Algebraic Functions.....	48
3.4	Sequences .....	55
3.5	Basic Limit Theorems .....	62
3.6	Limit Points .....	70
3.7	Special Sequences .....	74
3.7.1	Monotone Sequences .....	75
3.7.2	Convergence to Infinity .....	77
3.7.3	Cauchy Sequences .....	82
<b>4</b>	<b>Continuous Functions.....</b>	<b>87</b>
4.1	Limits of Functions.....	87
4.2	Continuity .....	89
4.3	Properties of Continuous Functions.....	94
4.4	Continuity of Special Functions .....	98
4.5	Uniform Continuity .....	104



<b>5</b>	<b>Differentiable Functions .....</b>	<b>107</b>
5.1	A Derivative of a Function .....	107
5.2	Basic Properties of Differentiable Functions.....	115
5.3	Derivatives of Special Functions.....	125
5.4	Higher Order Derivatives; Taylor's Theorem.....	131
5.5	L'Hospital's Rules.....	141
<b>6</b>	<b>Integration.....</b>	<b>147</b>
6.1	The Riemann Integral.....	147
6.2	Integrable Functions.....	155
6.3	Basic Properties of the Riemann Integral.....	158
6.4	The Fundamental Theorem of Calculus.....	166
6.5	The Mean-Value Theorems.....	171
6.6	Methods of Integration.....	175
6.7	Improper Integrals.....	179
<b>7</b>	<b>Infinite Series .....</b>	<b>183</b>
7.1	Convergence.....	183
7.2	Tests for Convergence.....	186
7.3	Conditional and Absolute Convergence.....	193
7.4	Multiplication of Series and Infinite Products.....	203
7.5	Power Series and Taylor Series.....	210
<b>8</b>	<b>Fourier Series.....</b>	<b>217</b>
8.1	Trigonometric Series.....	217
8.2	Convergence.....	225
8.3	Even and Odd Functions.....	229
8.3.1	Even Functions.....	229
8.3.2	Odd Functions.....	230
<b>9</b>	<b>Elementary Numerical Methods .....</b>	<b>233</b>
9.1	Introduction.....	233
9.2	Iteration.....	235
9.3	The Newton - Raphson Method.....	242
9.4	Interpolation Methods.....	247
9.4.1	Lagrange Polynomial.....	247
9.4.2	Cubic Splines.....	250
9.5	Least – Squares Approximations.....	252
9.5.1	Linear Least – Squares Method.....	253
9.5.2	Quadratic Least – Squares Method.....	254
9.6	Numerical Integration.....	256
9.6.1	The Trapezoidal Rule.....	256
9.6.2	Simpson Rule.....	257
9.6.3	Gaussian Integration.....	259

**10 Special Topics.....263**

    10.1 The Irrationality of  $e$  .....263

    10.2 Euler's Summation Formula .....264

    10.3 Lagrange Multipliers .....270

        10.3.1 Introduction: Multi-variable Functions .....270

        10.3.2 Lagrange Multipliers.....274

**Solutions to Selected Problem.....283**

**Index .....297**