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Quantum Theory of Near-Field Electrodynamics

With 33 Figures

 Springer

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To my wife Kaisa and son Sune

Preface

Within the last quarter of a century a new subfield of electrodynamics named *near-field optics* has emerged. In near-field optics, the light–matter interaction on a subwavelength scale is in focus. In the context of the electrodynamics of mesoscopic objects and condensed-matter media of macroscopic size the local-field concept thus is of central importance. Notwithstanding the fact that the overwhelming majority of the experimental studies up to now have dealt with problems in the visible, near-infrared, and near-ultraviolet regions of the electromagnetic spectrum, much of the basic theory covers a broader frequency spectrum. In view of this, I have preferred to use the term *near-field electrodynamics* in the title of the book.

In a sense, all the quantum theory of near-field electrodynamics is contained in the framework of quantum electrodynamics (QED), but, and this is the important point, studies in near-field electrodynamics make it possible for us to see certain aspects of QED in a new perspective. QED is the covering theory of semiclassical electrodynamics in which the electromagnetic field is treated classically and the particle dynamics by quantum mechanics (in its first- or second-quantized form). The semiclassical theory in itself is the covering theory of classical electrodynamics, in which the motion of the individual charged particles is determined by the Newton–Lorentz equation.

It has not been the purpose of this book to cover the many specific theoretical problems in near-field electrodynamics which physicists have been engaged in over the years. Instead, I have sought to give the reader an account of the basic theory, including the theory’s connection to electrodynamics as such. Even with this limited goal a number of important fundamental issues necessarily cannot be covered in a single volume. With the aim of placing the theory of near-field electrodynamics in its proper framework it is unavoidable not to touch upon themes covered in books on general electrodynamics. However, where possible I have attempted to describe such themes with an eye to the near-field perspective. Although many (and perhaps most) of the theoretical studies in near-field electrodynamics have been carried out within the framework of macroscopic (classical) electrodynamics, I have always held the point of view that a proper understanding of field–matter interaction on a subwavelength scale in most cases requires quantum physics. The content of the book reflects this standpoint, and there is no doubt in my mind that the quantum theory of near-field electrodynamics, and in particular its field-quantized version,

will take up a central place in the future. The quantum theory of near-field electrodynamics also influences our view of old subjects and problems, e.g., the spatial localization and confinement theory of transverse photons, a theory which links with the question of diffraction limitation in optics.

In a way, the central topic in near-field electrodynamics is the interplay between radiative and non-radiative (matter-attached) fields. This interplay takes place in a narrow region in the vicinity of matter. I have called this region the rim zone of matter. Theoretical studies of rim-zone electrodynamics appear fascinating and complicated, fascinating because they offer us a fresh view on electrodynamics (optics), and complicated due to the fact that the electromagnetic coupling between objects in rim-zone contact is strong. In the rim zone the irrotational part of the electric field, \mathbf{E}_L , is divergence-free, and the criterion $\nabla \cdot \mathbf{E}_L = 0$ I take as a definition of what is meant by “near” in near-field electrodynamics.

We begin the book (Part I) with elements of the microscopic classical theory of electrodynamics, originating in the works of J.C. Maxwell and H.A. Lorentz (and others). In this formulation matter is considered to consist of point-particles. Among other things, the Liénard–Wiechert fields, radiation reaction, multipole electrodynamics, point–dipole interactions, and global and local conservation laws are studied. Electromagnetic Green functions for the electric and magnetic fields play an important role in near-field electrodynamics, and these functions are treated in detail in various representations. Particular emphasis is devoted to the polar and angular spectrum representations. The last representation is of great importance in relation to studies of evanescent electromagnetic fields, and electromagnetic surface and interface waves.

In Part II, we discuss the semiclassical theory of near-field electrodynamics. Here, quantum mechanics is used to describe the particle dynamics, but the electromagnetic field is treated classically. Starting from considerations related to the division into transverse and longitudinal electrodynamics, we go on with a detailed account of the linear nonlocal response theory. This theory forms the basis for local-field calculations in electrodynamics. On the basis of the Liouville equation for the (many-body) density matrix operator the quantum theory of the so-called generalized nonlocal linear response is established. After accounts of the microscopic Ewald–Oseen extinction theorem, my own coupled-antenna theory, transverse and covariant electromagnetic propagators, and principal volume and self-field dynamics, follows chapters devoted to photon wave mechanics (PWM), i.e., the first-quantized theory of the photon. The covariant four-potential formulation of PWM is of particular importance in near-field electrodynamics, because the so-called longitudinal and scalar photons together only affects the physics in the rim zone. Although the covariant covering theory of PWM, namely QED, traditionally has been used in high-energy physics, I advocate the use of the covariant description in near-field electrodynamics, among other things, because it offers us a fresh view on the spatial localization problem for transverse photons emitted from single atoms and mesoscopic objects. It is described why superlocalization in space (spatial confinement to atomic dimension) may occur for the electromagnetic field in the initial moments of its emission from a pure spin transition. A short treatment of

one-particle position operators and spatial localization for massive particles helps one to understand that the lack of perfect spatial localization is shared by the transverse photon and the electron.

Albert Einstein's 1905 analysis of the high-frequency part of Max Planck's black-body radiation law indicated that electromagnetic fields at high frequencies may be considered as consisting of particles (later named photons). With this in mind we discuss the possibilities for establishing an eikonal theory for transverse photons. The limitation on this possibility relates to basic properties in near-field electrodynamics.

In Part III, the relation between QED and near-field electrodynamics is discussed. Due to the importance of Green function and electromagnetic propagator formalisms in rim-zone electrodynamics it is useful to extend the classical Maxwell–Lorentz equations to the operator level. In their quantized form these equations describe the time evolution of the global field–matter system in the Heisenberg picture. The formal form of the equations depends on the choice of gauge. In the Coulomb gauge the set of operator equations becomes form-identical to the classical Maxwell–Lorentz equations, whereas they in the Poincaré gauge take a different form. The Poincaré gauge relates to a specific choice for the generalized polarization and magnetization fields, quantities of substantial importance in the classical description of near-field interactions.

A careful study of field commutator relations, whether it be for free fields or fields in the presence of matter, is a necessity in QED, and the connection of various field commutators to the possibilities for measuring electromagnetic fields in space-time was discussed in detail already in 1933 by Niels Bohr and Leon Rosenfeld. In the wake of the renormalization program developed around 1947 to deal with the problem of infinities in QED, these authors gave a more exhaustive treatment of the question of field and charge measurements in quantum electrodynamics in 1950. In the context of near-field electrodynamics, the commutator between the transverse parts of the vector potential and the electric field turns out to be of particular interest because this commutator in a quite direct manner expresses our inability to localize photons in space beyond the extension of the rim zone. The study of the field commutator relations leads one to integral representations of various covariant scalar propagators, most notably the Jordan–Pauli propagator and the Feynman meson propagator.

The QED description of the nonrelativistic particle–field interaction is studied in some generality and via examples of particular interest in near-field optics and mesoscopic electrodynamics. The link between QED and PWM is established by acting on single-photon wave packet states with certain photon-field operators which relate to the two photon helicity species. The understanding of the photon emission from single atoms (and mesoscopic particles) is of central importance in near-field QED, and we study this emission in some detail for a so-called two-level atom. From the dynamical equations for the coupled two-level atom plus field system a qualitative description of the spontaneous emission process and the Lamb shift can be obtained. Examinations of the particle–particle interaction via exchange of transverse photons lead us to qualitative theories for (1) the delay and magnetic

corrections to the Coulomb interaction between two charged particles, (2) the van der Waals interaction between two electrically neutral particles (or mesoscopic objects), and (3) the Casimir theory for particle–surface interactions.

A study of matter-attached quantized fields is central to near-field QED, and we approach this subject by a manifestly Lorentz-covariant photon description. The covariant quantization of the four-potential raises some fundamental problems, which solution brings us in contact with the Gupta–Bleuler–Lorenz condition and field quantization with an indefinite metric. Subsequently, some efforts are devoted to an introduction of the near-field plus gauge photon concept in QED, and to a discussion of the covariant Feynman photon propagator. Recently, the transverse part of this propagator has turned out to be of importance in connection to studies of field correlations in near-field photon wave mechanics, when this is based on photon position states defined via the transverse part of the vector potential.

Matter-attached fields are unavoidably present in the near-field zone of matter, and in the covariant notation their quantization leads to the scalar and longitudinal photons, and then by a certain unitary transformation to gauge and near-field photons. The conceptual usefulness of the matter-attached field quantization is illustrated by two examples: (1) The interaction between two fixed charges and (2) the radiation of scalar and longitudinal photons from a classical sheet current with negligible quantum fluctuations. Example (1) leads, as it is well-known to a reinterpretation of the Coulomb interaction as being due to an exchange of scalar photons between the two charges. Example (2) obviously is of interest for studies of evanescent fields in QED.

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Aalborg
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Ole Keller

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